Discount-Rate Risk in Private Equity: Evidence from Secondary Market Transactions*

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ABSTRACT

Measures of private equity performance based on cash flows do not account for a discount-rate risk premium that is a component of CAPM alpha. We create secondary market PE indices and find that PE discount rates vary considerably. NAVs are too smooth because they fail to reflect variation in discount rates. While the CAPM alpha for our index is zero, the GPME based on cash flows is large and positive. We obtain similar results for a set of synthetic funds that invest in small cap stocks. Ignoring variation in PE discount rates can lead to a misallocation of capital.

In recent decades, private equity (PE) has become an important asset class for investors. A recent survey of institutional investors finds that 88% are invested in PE, with nearly a third having an allocation greater than 10% (Whyte (2017)). In the early 2000s, a secondary market developed in which limited partners (LPs) can transact their stakes in PE funds.[1] Since data on secondary market prices are not publicly available, investors and academics have been limited to using data on cash flows to evaluate the risk and return of PE. As much as half of the variation in *public* equity returns, however, is driven by news about discount rates (Campbell (1991). Further, all variation in dividend price ratios for public equity corresponds to news about discount rates rather than news about cash flows (Campbell and Shiller (1988), Cochrane (2011)). These empirical facts for public equity naturally lead to two questions about PE investments that we address in this paper. First, exactly how might discount-rate risk create disparity between cash-flow-based measures of investment performance and standard measures, such as the capital asset pricing model (CAPM) alpha? Second, to what extent does discount-rate risk in PE generate a meaningful empirical difference between the two types of investment performance measures?

To answer the first question, we derive the relation between the generalized public market equivalent (GPME) of Korteweg and Nagel (2016) and alpha when both are measured relative to the same stochastic discount factor (SDF). We show that alpha accounts for a discount-rate risk premium that separates the two measures. One way to understand the difference between alpha and GPME is to note that alpha characterizes risk in terms of a *total-return* beta, while GPME characterizes risk in terms of *cash-flow-yield* betas that ignore any covariation between the SDF and market prices or discount rates.

Turning next to the second question, we investigate the extent to which discount-rate risk in PE leads to a meaningful empirical difference between GPME and CAPM alpha. To do so we use data obtained from a large intermediary to construct market-based indices for buyout funds that enable us to measure the

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[1] While secondary market liquidity was limited in the early years of the secondary market, liquidity has grown substantially over the past decade.

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CAPM alpha of PE. In contrast to much of the existing literature that investigates PE performance using cash flows, our alpha estimates suggest that buyout funds do not outperform public markets on a risk-adjusted basis.[2]

In constructing our PE indices we carefully address the potential problem of sample selection. For our defined universe of funds, no fund trades in every period, and many funds do not trade at all. Moreover, funds that do trade are not likely to be randomly chosen. In the subsample of funds that can be matched with cash flow data from the *Preqin* database, there are 839 buyout transactions for 287 unique funds over the period 2006 to 2018, implying that the average fund trades 2.9 times in our sample conditional on trading at all. To explicitly account for possible sample selection in the set of funds for which we observe transactions, we use the approach of Heckman (1979) and estimate the parameters of an econometric model using observed transaction prices. We use this model to estimate the total index return over our sample period for a defined population of funds.[3] When estimating index parameters, we account for non-synchronicity in prices using the approach of Dimson (1979).

We estimate the total-return beta of our main secondary market PE index to be 1.79. As Axelson, Sorensen, and Strömberg (2014) emphasize, the return on a buyout fund is effectively the return on a portfolio of highly levered firms. If the portfolio firms prior to the buyout have unlevered betas around 1.0, then doubling their leverage, as was typically done in buyouts during our sample period, should lead to a portfolio beta of around 2.0.[4] The benchmark beta of 1.0 in this calculation is a total-return beta, in contrast to the cash-flow-yield betas implicit in cash-flow measures of performance. This basic calculation therefore

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[2] For example, current evidence suggests that buyout funds outperform on a risk-adjusted basis (Higson and Stucke (2012), Harris, Jenkinson, and Kaplan (2014), and Robinson and Sensoy (2013)).

[3] Other indices based on secondary markets incorporate some type of interpolation to infer the prices of nontraded assets. Bond indices, for instance, often employ "matrix pricing" to infer the prices of nontraded bonds.

[4] This calculation assumes that the debt beta equals zero. In fact, estimates of buyout debt betas are positive, which would lead the fund-level beta in this example to be less than two (see Kaplan and Stein (1990)).

produces a total-return beta for PE that is consistent with the total-return betas we estimate using our indices. For our main index, we document an annual alpha of -2% that is insignificantly different from zero. As mentioned above, this finding contrasts with most of the literature that finds buyout funds outperform public markets on a risk-adjusted basis using cash-flow measures of performance. While we find the beta of a listed PE index to be very similar to the betas of our transaction-based PE indices, the average return and alpha of the listed PE index are considerably lower. These findings highlight the difficulty of listed PE indices to accurately capture the performance of LP investments.

To further investigate the second question above, we estimate time-series variation in PE discount rates. We find that PE log book-to-market ratios vary considerably for our secondary market PE index. Using regressions motivated by the Campbell-Shiller (1988) identity, we also find that all variation in log book-to-market ratios for PE can be explained by variation in market discount rates. A similar phenomenon has been documented for other asset classes including public equities, treasuries, credit, foreign exchange, and sovereign debt.[5] From these regressions we find the standard deviation of long-run PE discount rates to be about 0.36. Cochrane (2011) estimates the standard deviation of discount rates at similar horizons for public equity to be 0.29. Our index, however, is highly levered with a beta of 1.79 whereas Cochrane's results correspond to a unit-beta portfolio. Hence, a more appropriate comparison from public markets might be the standard deviation of discount rates for a levered market portfolio with a beta of 1.79, that is, $1.79 \times 0.29 = 0.52$. From this perspective, PE discount rates may vary somewhat *less* than public market discount rates. Dynamics in risk-aversion and sentiment for the marginal investor in PE may be somewhat muted relative to that of the marginal investor in public equity. In contrast, the standard deviation of book, or net asset value (NAV) discount rates is only 0.10. NAVs are too smooth from the perspective of an investor with access to secondary markets, not only because they reflect stale information (e.g., Ewens, Jones, and Rhodes-Kropf (2013)), but also because they fail to reflect variation in market discount rates for PE.

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[5] Cochrane (2011) provides a summary of the literature.

To directly address the second question above, we next compare estimates of GPME for funds in our index with the index CAPM alpha. In contrast to our index alpha, the GPME for funds in our main index is relatively large at about 0.27 and is comparable to that of venture funds pre-1998, a period of strong performance for venture funds (Korteweg and Nagel (2016)). While our empirical results suggest that a positive discount-rate risk premium creates a wedge between GPME and alpha, we lack power to estimate GPME with much precision using the overlapping cash flows of the funds in our universe. In addition, our results may be driven by specific features of PE secondary markets, transaction costs, or the manner in which we construct our PE secondary market indices. To provide further evidence that discount-rate risk can separate cash-flow-based measures of performance from standard measures, we estimate GPME and alpha using a group of investments in public companies for which valuations and portfolio construction are transparent and transaction costs are minimal. In particular, we construct the cash flows for a series of synthetic funds that invest in size-decile portfolios of public equities. For these synthetic funds we find the estimated GPMEs to be large and highly statistically significant, but CAPM alphas to be virtually zero, insignificant, and, in some cases, slightly negative. These results provide additional support for the view that discount-rate risk can separate the two types of investment performance measures.

What kind of investor cares about variation in discount rates? The utility of any investor with some positive probability of accessing PE secondary markets may be impacted by variation in PE discount rates. Many PE investors, however, do not engage in PE secondary markets and simply collect the cash flows. In reality, all LPs may access PE secondary markets but face costs and constraints, including the potential cost of harming relations with general partners (GPs), that prevent them from doing so. It is similarly common for investors who buy corporate bonds to avoid trading in secondary markets and collect the coupons. Many bond investors, however, dynamically optimize and rebalance their portfolios in markets for credit and interest rate swaps, which are facilitated by the existence of secondary market prices for bonds. Just as the factor exposures of coupon payments do not capture the full risk exposure of bond investors, the factor exposures of PE cash flows do not capture the full risk exposure of PE investors with a variety of myopic

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or dynamic objectives. There exist states of the world in which even self-proclaimed "buy-and-hold" investors will find it optimal to engage in PE secondary markets, as we observed during the financial crisis.[6]

Finally, what role do transaction costs such as liquidity discounts play in our results? The returns of all asset classes in secondary markets are influenced to some degree by transaction costs. As a seller-driven market, transaction costs in PE secondary markets appear to be borne by sellers. Buyers earn a small premium as market makers if positions are held to the end of a fund's life (Nadauld et al. (2019, NSVW)). Our index returns, however, reflect returns from buying *and* selling at secondary market prices. To the extent that buyers earn a premium for market making, this premium is largely canceled out when the position is sold three months later.[7]

Our paper is organized as follows. In Section I we discuss various approaches the literature has proposed to measure risk in PE. Section II derives the relation between GPME and alpha. Section III describes the methodology we use to create a PE index using secondary market prices. Section IV summarizes our data. Section V presents estimates of the model used to create the index and subsequent PE alphas, betas, and other performance metrics arising from the index. Section VI investigates variation in PE discount rates and compares estimates of GPME and CAPM alpha for funds in our index and a set of synthetic funds that invest in public equities. Section VII concludes.

I. Prior Work Measuring Private Equity Risk and Return

For most asset classes, investors rely on secondary market transaction-based indices to measure performance, in which case risk can be appropriately characterized as the covariance of the total return with

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[6] Miller, Clair Cain, and Geraldine Fabrikant, 2008, Universities retrench as endowments suffer from financial crisis, *New York Times*, October 25.

[7] To the extent that the liquidity discount is slowly decreasing over time as markets become more liquid during our sample, estimated performance measures based on transaction prices, such as alpha, represent an upper bound. Estimates of CAPM alpha based on our index suggest that buyout funds do not outperform public markets on a risk-adjusted basis, and any accounting for a decreasing liquidity premium only strengthens this result.

a relevant SDF. Because such secondary markets did not exist for many years in PE, alternative approaches were developed to measure and assess risk and return in this market. Korteweg and Nagel (2016) provide a thorough survey of the PE performance literature. Here we review a few highlights. We classify prior studies of PE investment performance into six groups that vary depending on the recommended approach.

First, a number of studies estimate the CAPM alpha and beta of PE using book returns based on the NAV specified by the GP (see Anson (2013), Woodward (2009), and Ewens, Jones, and Rhodes-Kropf (2013)). Because the portfolio companies often have no observable market price, marked values often do not reflect the most recent information in public markets. To accurately measure systematic risk, these authors project book returns on current and lagged market returns and then sum up the estimated coefficients to obtain an estimator for beta as in Dimson (1979). These studies find beta for buyout companies to be in the range of 0.70 to 1.0, with alphas ranging from 3.2% to 4.8% annualized.

A second group of studies evaluate PE performance by estimating the public market equivalent (PME) of Kaplan and Schoar (2005) using cash flows paid to and received by LPs. Recent studies that use net-of-fee, fund-level, cash-flow data find the PME for buyout funds to be in the range of 1.19 to 1.23, suggesting that buyout funds outperform public equity markets even after adjusting for fees (see Higson and Stucke (2012), Harris, Jenkinson, and Kaplan (2014), and Robinson and Sensoy (2013)). Kortegweg and Nagel (2016) develop the GPME to relax restrictions on the SDF implicit in the PME and study the performance of venture companies.[8]

A third group of studies evaluate PE performance by estimating factor alphas and betas from regressions of log internal rates of return (IRRs) on the contemporaneous IRRs of the public equity market

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[8]Whereas the PME is the *ratio* of discounted distributions to discounted capital takedowns, the *GPME* is the *difference,* or net present value (NPV) based on an SDF. When capital calls are stochastic, asset pricing theory makes no clear predictions about the expected ratio because of a Jensen's inequality effect (see Kortegweg and Nagel (2016)). If we define the PME as discounted distributions *minus* contributions, then the GPME nests the PME with the SDF defined as the inverse public market return.

and other factor portfolios. Most papers that take this approach estimate fund log IRRs from cash flows between PE funds and their portfolio firms (see Frazoni, Nowark, and Phalippou (2012) and Axelson, Sorensen, and Strömberg (2014)). These papers find buyout betas to be in the range of 0.90 to 2.4 with alphas in the range of 8.6% to 9.3%.[9]

A fourth group of studies evaluate the performance of PE by creating mimicking portfolios of publicly traded securities that are similar to PE in terms of portfolio company characteristics (Groh and Gottschalg (2011) and Stafford (2022)). In a related approach, Gupta and Van Nieuwerburgh (2021)) create synthetic strips to match the cash flows paid to and from LPs using a variety of exchange-traded securities. The market prices of these strips cannot be observed and are evaluated using a no-arbitrage pricing model. These studies find buyout betas in the range of 1.4 to 1.8 and alphas in the range of 8.4% to 11.8% annualized.

A fifth group of studies develop methods to estimate PE buyout risks by imposing a model on systematic cash-flow risks across funds and over time and incorporating pricing information from public markets (Buchner and Stucke (2014), Driessen, Lin, and Phalippou (2012), and Ang et al. (2018)). These papers find buyout betas in the range of 1.3 to 2.7 and alphas from -4.8% to 4.5% annualized. To the extent that discount rates in public markets are correlated with those in PE markets, we might expect such approaches to be influenced to some degree by the discount rate effects we document.

Finally, a sixth group of studies develop approaches to evaluate PE based on the returns to publicly traded PE securities (Jegadeesh, Kraussl, and Pollet (2015) and McCourt (2018)). These studies estimate betas for buyout funds to be in the range of 0.7 to 1.1 with alphas from 1.2% to 7.2% annualized.

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[9] In contrast to our work, studies based on cash flows between PE funds and their portfolio firms estimate risk and return *gross of fees*. Carried interest, which is similar to a short call position from the perspective of the investor, causes the net-of-fee beta to be lower than the gross-of-fee beta if the fund itself covaries positively with the market. Kaplan and Schoar (2005) run similar regressions using IRRs estimated from cash flows paid to and received by LPs.

Overall, the vast majority of evidence in the literature suggests that buyout funds tend to outperform on a risk-adjusted basis using data on PE cash flows. But valuation and performance measurement based on cash-flow data can, in general, account only for *cash-flow* risk exposures. Factor exposures measured in much of the existing work are "cash-flow betas" that measure the covariance between the SDF and cash flows of the underlying PE investment.[10] These estimates miss risk exposures coming from co-movement of the SDF with valuations associated with changes in PE discount rates. Without data on actual secondary market prices, it is impossible to understand the magnitude of this risk and the impact it may have on PE performance and standard metrics such as alpha and beta.

Some studies evaluate PE performance using either PE NAVs or exchange-traded asset returns. The well-known excessive "smoothness" of NAVs has been attributed to the use of stale information by GPs. But the impact of stale information can be accounted for using standard statistical techniques such as those proposed by Dimson (1979). Any excess smoothness in NAVs after appropriately accounting for the use of stale information should arise from other causes. We provide evidence that variation in NAVs fails to account for dynamics in market PE discount rates.

To the extent that discount rates are correlated, market prices of other exchange-traded products could reflect some of the variation in PE discount rates, but other assets are clearly not perfect substitutes for PE. The marginal investor in PE is likely to differ from the marginal investor for other asset classes, such as public stocks, in terms of preferences and sophistication. Dynamics in sentiment and risk-aversion and their impact on discount rates may differ for PE than for other asset classes. Moreover, the organizational structure of PE may also generate unique properties in PE risk.

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[10]They are cash-flow betas in a different sense than in Campbell and Vuolteenaho (2004). These authors define the cash-flow beta as the covariance between the *total return of the asset* being evaluated and the *component of the SDF attributable to news about aggregate cash flows*. In contrast, the factor exposures and SDF covariances behind performance measures using PE cash flows in the extant PE literature represent covariances between the *cash flows of the asset* being evaluated and the *total SDF*.

Listed PE securities, in particular, mostly represent publicly traded funds of funds and the equity shares of GPs that have listed publicly. Their returns are likely to be correlated with those received by LP returns in PE funds but are also likely to be different for at least four reasons. First, the cash flows of publicly traded securities of PE firms reflect cash flows of the GPs, whose claim is the present value of future fees and carried interest earned by the fund, rather than the cash flows of the LP in a particular fund. Second, large, publicly traded buyout firms such as Blackrock and KKR hold a variety of investments other than PE, including hedge funds, real estate, advisory services, etc. Third, some of the publicly traded PE funds are funds of funds that charge an extra layer of fees (that varies with performance) in addition to the fee collected by the managers of the unlisted funds in which funds of funds invest. Finally, there is potential for sample selection in the types of funds that choose to list public shares.

II. Performance Evaluation Based on Cash Flows versus Transaction Prices

In this section we present a conceptual framework that connects cash-flow valuation measures with traditional valuation measures in asset pricing. In doing so, we characterize the way in which an investor might use information in secondary market prices to make capital allocation decisions.

A. Two First-Order Conditions

The standard Euler equation that defines optimal allocation to asset i is given by

$$E_t[M_{t:t+1}(X_{i,t+1} + p_{i,t+1})] - p_{i,t} = 0, \tag{1}$$

where $X_{i,t+1}$ is the cash flow paid by the asset at time $t + 1$, $p_{i,t+1}$ is the market value of the asset at time $t + 1$, and $M_{t:t+j}$ is discounted growth in marginal utility from t to $t + j$,

$$M_{t:t+j} = \delta^j \frac{u'(c_{t+j})}{u'(c_t)}, \tag{2}$$

with δ denoting the subjective discount factor. The left side of equation (1) is the one-period conditional NPV from investing in asset i,

$$NPV_{i,t} = E_t[M_{t:t+1}(X_{i,t+1} + p_{i,t+1})] - p_{i,t}, \tag{3}$$

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which measures the marginal utility gain from an incremental investment in the asset for an investor with access to secondary markets. This interpretation of NPV is analogous to the standard interpretation of performance measures in the mutual fund and hedge fund literatures. The characterization of $M_{t:t+j}$ may be investor-specific and does not necessarily reflect an equilibrium pricing kernel. We refer to $M_{t:t+j}$ as the investor's SDF. After dividing both sides of (3) by p_t, the NPV becomes the conditional alpha of asset i relative to $M_{t:t+1}$.

Now consider a long-term buy-and-hold investor who cannot access secondary markets for asset i. At time t the investor chooses the number of additional units of the asset to acquire, where each unit provides a series of stochastic cash flows, $X_{i,t+j}$, that arrive at times $t+j$ for $j = 1,..T$. If the investor makes this choice to maximize the expected additive utility of consumption over T periods, then the first-order condition, subject to standard budget constraints, is

$$E_t \sum_{j=1}^{T-t} X_{i,t+j} M_{t:t+j} - p_{i,t} = 0. \tag{4}$$

The left side of equation (4) is the conditional GPME of Korteweg and Nagel (2016),

$$GPME_{i,t} = E_t \sum_{j=1}^{T-t} X_{i,t+j} M_{t:t+j} - p_{i,t}, \tag{5}$$

which measures of the marginal utility gain from an incremental investment in the asset for an investor with no access to secondary markets.

If the investor's SDF represents an equilibrium pricing kernel, then the two valuation measures are equivalent and equal to zero. If the SDF is investor specific, however, GPME and NPV may diverge. An investor with no access to secondary markets may value the asset differently than an investor that does have access, even if the two investors are otherwise identical. We further illustrate from two perspectives.

First, NPV clearly depends on the covariance of secondary market prices with the SDF, while GPME does not. For instance, when $M_{t:t+j}$ is represented specifically by a CAPM SDF, as in Korteweg and Nagel (2016), NPV depends directly on the covariance of secondary market prices for asset i with the

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market portfolio. In this case, NPV measures the marginal utility gain from an incremental investment in the asset for a myopic one-period investor who will sell the asset at the end of the period and currently holds positions in a stock market fund and risk-free bonds. In contrast, GPME measures the marginal utility gain from an incremental investment for a buy-and-hold investor who simply collects the cash flows until the fund expires, and otherwise holds positions in a stock market fund and risk-free bonds. Since the buy-and-hold investor never participates in secondary markets for asset i, GPME is immune to secondary market prices. As such, transitory price fluctuations for asset i that arise from variation in market discount rates for the asset's cash flows can generate risks that make the NPV higher or lower than the GPME, depending on whether such price movements are negatively or positively correlated with the stock market return.

We can directly illustrate this distinction between NPV and GPME by writing both valuation measures in terms of pre-determined discount rates. The implicit discount rate for NPV depends on the *total- return beta*, while the implicit discount rates for GPME depend on *cash-flow-yield betas*. In Appendix A we show that NPV can be expressed as the present value of one-period cash flows minus the current price,

$$NPV_{i,t} = \frac{E_t\left(X_{i,t+1} + P_{i,t+1}\right)}{K_{i,t}} - p_{i,t},$$

(6)

where $K_{i,t}$ is a discount rate that is a linear function of the *total return* beta, $\beta_{i,t}$, given by

$$\beta_{i,t} = \frac{Cov_t\left[M_{t:t+1}, R_{i,t+1}\right]}{Var_t\left(M_{t:t+j}\right)},$$

(7)

and $R_{i,t+1}$ represents the total return of asset i from t to $t+1$. In contrast, GPME is equivalent to the present value of all future expected cash flows minus the current price,

$$GPME_{i,t} = \sum_{j=1}^{T-t} \frac{E_t\left(X_{i,t+j}\right)}{\mathcal{K}_{i,t(t+j)}} - p_{i,t},$$

(8)

where the discount rate $\mathcal{K}_{i,t(t+j)}$ is a linear function of the cash-flow-yield beta, $\mathcal{B}_{it(t+1)}$, given by

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$$\mathcal{B}_{it(t+1)} = \frac{Cov_t[M_{t:t+j}, Y_{i,t+j}]}{Var_t(M_{t:t+j})} \tag{9}$$

and $Y_{i,t+j}$ is the cash-flow yield, or rather, the cash flow arriving at $t + j$, $X_{i,t+j}$, divided by its valuation. The total return beta depends on the covariance between secondary market prices and the SDF. In contrast, cash-flow-yield betas, which only account for the covariance between cash-flows and the SDF, do not. Further details can be found in Appendix A.

Second, for investors that can dynamically rebalance every period to maximize expected utility, the appropriate SDF to measure NPV will generally depend on the secondary market price for asset i while the appropriate SDF to measure GPME will not. Consider two investors that can dynamically rebalance in other assets every period (such as stocks and bonds) and that are considering a purchase of asset i. One investor can also access secondary markets for asset i while the other cannot, but the two investors are otherwise identical. In this case the SDF for the unconstrained investor will depend on optimal consumption, savings, and investment choices at time $t + 1$, which will generally depend on the prevailing secondary market price for asset i. In contrast, the SDF for the constrained investor is immune to the asset's secondary market price.

The expected utility of the unconstrained dynamic investor with secondary market access in this case must be at least as high as the expected utility of the constrained investor, since the buy-and-hold strategy is among the possible dynamic strategies the unconstrained investor can pursue. Moreover, if future cash flows from the asset are all positive, then the NPV must also be at least as high as the GPME. The current perceived value of an asset with positive cash flows that includes options to liquidate must be at least as high as an otherwise identical asset with no such options. However, if some future cash flows are negative, as they are for a PE deal, then NPV may be less than GPME even for investors that dynamically rebalance. Depending on the distribution of future secondary prices and future states, it may be optimal for the unconstrained investor to preserve the option to acquire a stake in the PE deal *later* at potentially favorable secondary prices while avoiding high capital calls that potentially arrive in bad states of the world.

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Regardless of the SDF or the nature of investors that invest in PE, the value of an incremental investment in PE for an unconstrained investor with access to secondary markets cannot be measured without secondary market prices. In this paper we make this valuation possible by creating a secondary market PE index. To connect our results to prior work, in the next subsection we derive the relation between unconditional GPME and unconditional alpha when the same SDF is used to measure both, and in our empirical work we use a CAPM SDF to compare unconditional GPME and alpha. We leave an empirical investigation of the dynamic portfolio selection problem when secondary markets are accessible for future research.

B. GPME and Alpha

In this section we derive the mathematical relation between GPME and alpha when the same SDF is used to measure both. Assume that a PE fund pays cash flows, $X_{i,t}$, beginning at $t = 1$ and ending at $t = T$, where cash flows can be positive, negative, or zero. Define the unconditional GPME of the fund as

$$GPME_i = E\left[\sum_{j=0}^{T-t-1} M_{t:t+j+1}X_{i,t+j+1} - p_{i,t}\right]. \tag{10}$$

We can add and subtract $M_{t:t+j+1}p_{t+j+1}$ to the term inside the expectations operator,

$$GPME_i = E\left[\sum_{j=0}^{T-t-1} M_{t:t+j+1}X_{i,t+j+1} + M_{t:t+j+1}p_{i,t+j+1} - M_{t:t+j+1}p_{i,t+j+1} - p_{i,t}\right]$$

$$= E\left[\sum_{j=0}^{T-t-1} M_{t:t+j+1}(X_{i,t+j+1} + p_{i,t+j+1}) - M_{t:t+j}p_{i,t+j}\right], \tag{11}$$

where the second line of (11) follows since $M_{t:t}p_{i,t} = p_{i,t}$ and $p_{i,T} = 0$. But then we can factor out $M_{t:t+j}$ on the right side of (11),

$$GPME_i = E\sum_{j=0}^{T-t-1} M_{t:t+j}[M_{t+j:t+j+1}(X_{i,t+j+1} + p_{i,t+j+1}) - p_{i,t+j}]. \tag{12}$$

By iterated expectations, we have

$$GPME_i = E\sum_{j=0}^{T-t-1} M_{t:t+j}E_{t+j}[M_{t+j:t+j+1}(X_{i,t+j+1} + p_{i,t+j+1}) - p_{i,t+j}], \tag{13}$$

which, from equation (3), implies

$$GPME_i = E\left[\sum_{j=0}^{T-t-1} M_{t:t+j}NPV_{i,t+j}\right]. \tag{14}$$

The unconditional GPME is equivalent to the expected sum of stochastically discounted conditional NPVs over the life of the fund.

Conditional alpha is simply $NPV_{i,t+j}$ scaled by price,

$$\alpha_{i,t+j} = NPV_{i,t+j}/p_{i,t+j}, \tag{15}$$

implying that we can write equation (14) as

$$GPME_i = E\left[\sum_{j=0}^{T-t-1} M_{t:t+j}p_{i,t+j}\alpha_{i,t+j}\right]. \tag{16}$$

This implies in turn that

$$GPME_i - \sum_{j=0}^{T-t-1} Cov(M_{t:t+j}\,p_{i,t+j},\,\alpha_{i,t+j}) = \alpha_i \sum_{j=0}^{T-t-1} E[M_{t:t+j}p_{i,t+j}], \tag{17}$$

where α_i is the expected conditional alpha. Given a strictly positive SDF, the summation term on the right of (17), which represents the present value of the sum of expected future secondary prices, is positive. The right side of (17) therefore takes the same sign as α_i and is zero if and only if $\alpha_i = 0$. On the left side of (17) we have the unconditional GPME minus a risk premium.

To provide intuition for the risk premium in equation (17), note that $M_{t:t+j}p_{t+j}$ can be interpreted as the (scaled) marginal benefit of selling and consuming an incremental unit of the PE stake in secondary markets at time $t + j$.[11] Further, conditional alpha is equivalent to a ratio of discount rates. To see this equivalence, let M^*_{t+j} denote an equilibrium pricing kernel. Conditional alpha can then be written as

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[11] Define consumption at time $t + j$ as $c_{t+j} = e_{t+j} + z_{t+j}p_{t+j}$, where z_{t+j} denotes the amount of the PE stake sold and consumed, and e_{t+j} represents consumption when z_{t+j} is equal to zero. Then $\partial u(c_{t+j})/\partial z_{t+j} = u'(c_{t+j})p_{t+j} = [u'(c_t)/\delta^j]M_{t:t+j}p_{t+j}$.

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$$\alpha_{i,t+j} = \frac{NPV_{i,t+j}}{p_{i,t+j}} = \frac{E_{t+j}[M_{t+j+1}(X_{i,t+j+1} + p_{i,t+j+1})]}{E_{t+j}[M^*_{t+j+1}(X_{i,t+j+1} + p_{i,t+j+1})]} - 1. \tag{18}$$

In Appendix A we show that

$$E_t[M_{t:t+1}(X_{i,t+1} + p_{i,t+1})] = \frac{E_t[X_{i,t+1} + p_{i,t+1}]}{K_{i,t}}, \tag{19}$$

where $K_{i,t}$ is a pre-determined discount rate known at time t that is a linear function of the total-return beta of the PE stake relative to the SDF, given in equation (7). Equations (18) and (19) imply that conditional alpha is simply equal to a ratio of discount rates,

$$\alpha_{i,t+j} = \frac{K^*_{i,t+j}}{K_{i,t+j}} - 1, \tag{20}$$

where $K^*_{i,t+j}$ is the market discount rate based on the pricing kernel and $K_{i,t+j}$ is the discount rate based on the investor-specific SDF. The risk premium on the left of (17) therefore represents a unique form of discount-rate risk that captures the covariance between the marginal benefit of selling and consuming an incremental unit of the PE stake and the prevailing market discount rate for PE cash flows. If the realized marginal benefit of consuming an incremental unit of the PE stake tends to be high when the market discount rate, $K^*_{i,t}$, is relatively high and prices are low, this positive covariance creates disparity between GPME and alpha. Alpha accounts for this discount-rate risk premium while GPME does not.

III. Creating an Index

Relative to creating indices on exchange-traded securities such as stocks, creating a market-based PE index is challenging because not every fund transacts every period and those that do transact are not selected randomly. Our approach to creating an index of PE returns relies on three key components: two first-order approximations and the estimation of a Heckman (1979) sample selection model. We first lay out the basic intuition of our approach. We then provide additional details.

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A. Intuition of Index Construction

To construct an index of returns, suppose we observe quarterly returns for N assets, $\tilde{r}_1, \ldots, \tilde{r}_N$. The equal-weighted portfolio return for the quarter, $\tilde{r}_p$, is the cross-sectional average,

$$\tilde{r}_p = \frac{1}{N}\sum_{i=1}^{N}\tilde{r}_i. \tag{21}$$

If we project returns on a set of characteristics, x_i, that includes an intercept,

$$\tilde{r}_i = x_i'b + e_i, \tag{22}$$

then we can recover the true realized equal-weighted portfolio return as

$$\tilde{r}_p = \bar{x}'b, \tag{23}$$

where $\bar{x}$ is the cross-sectional average value of x_i across all N funds. In the linear projection given by equation (22), both e_i and $x_i e_i$ average out exactly to zero across the N funds in our population. These residual properties hold, and we can recover the true realized portfolio return, for *any* x_i we choose, provided that x_i contains an intercept. The purpose of equations (22) and (23) is to calculate the realized portfolio return, not to estimate the parameters of a model we hope is relevant outside our population of N funds. Indeed, the standard approach to calculating realized portfolio returns given by equation (21) represents a special case of (23) in which x_i is a scalar and equal to one.

We do not observe $\tilde{r}_i$ for the full population of N funds, but rather only for a sample of funds in a given quarter. For any choice of x_i, if we were to estimate b using only the observations for which we have data on both $\tilde{r}_i$ and x_i, then the OLS estimate of b will be inconsistent unless the subset of funds that transact are chosen randomly. Nadauld et al. (2019) document, however, that larger and more established funds are more likely to transact in this market, and hence the funds that do transact do not appear to be randomly selected.

We therefore use a selection model to consistently estimate b, based on the approach of Heckman (1979). While we do not observe $\tilde{r}_i$ for every fund, we do observe x_i for every fund. Using our estimate of b we calculate the index return as the average predicted return each quarter as in (23) using *all N* funds in

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the population. We thus obtain consistent estimates of index returns each quarter based on the PE secondary market transactions we observe.

B. Inferring Index Returns from Book-to-Market Ratios

While the preceding discussion outlines the basic approach to index construction, we now explain further details. Funds do not often transact in adjacent quarters, and as such the number of individual fund transactions is much greater than the number of observed individual quarterly fund returns. To maximize our use of available data, we infer index returns from observed log book-to-market ratios in adjacent quarters using the Campbell-Shiller (1988) identity. Doing so enables us to take advantage of all transactions in our sample, regardless of whether the same funds transact in adjacent quarters or not. Besides enabling us to estimate returns for our PE index, our approach is convenient for understanding variation in PE discount rates, since all variation in log book-to-market ratios must be associated with variation in market discount rates, variation in book discount rates, or variation in future book-to-market ratios.

Let the log quarterly portfolio market return for PE fund i from t to $t+1$ be defined as

$$r_{i,t+1} = \ln\left(\frac{p_{i,t+1} + X_{i,t+1}}{p_{i,t}}\right), \tag{24}$$

where, as before, $p_{i,t+1}$ is the market value of the fund at time $t+1$, and $X_{i,t+1}$ represents total distributions minus capital calls for the portfolio from t to $t+1$.[12] Similarly, define the log-book return based on NAVs as

$$n_{i,t+1} = \ln\left(\frac{NAV_{i,t+1} + X_{i,t+1}}{NAV_{i,t}}\right), \tag{25}$$

where $NAV_{i,t+1}$ is the NAV (book value) of the fund at time $t+1$. Our approach relies on two first-order approximations. The first is similar to an approximation of Vuolteenaho (2002) based on the Campbell-Shiller (1988) identity and is further developed in Appendix B. This approximation provides a natural approach to adjust book returns to reflect market returns,

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[12] When merging cash flow data with market values, we normalize to a $1 commitment.

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$$r_{i,t+1} = n_{i,t+1} - \rho_{i,t+1}\,\theta_{i,t+1} + \theta_{i,t}, \tag{26}$$

where $\theta_{i,t}$ is the log book-to-market ratio of the fund,

$$\theta_{it} = \ln(NAV_{i,t}) - \ln(P_{i,t}), \tag{27}$$

and $\rho_{i,t+1}$ is an approximation parameter that can take one of three values. If $X_{i,t+1}$ is positive (negative), then $\rho_{i,t+1}$ is slightly below (above) one. If $X_{i,t+1}$ is zero, then $\rho_{i,t+1}$ is equal to one and (26) holds exactly. (See Appendix B for further details.) Book values are likely to be among the most informative variables that we can observe regarding the expected cash flows of the fund. Since we observe book returns for every fund in our index each period, our market-based returns, $r_{i,t+1}$, contain all pricing information embedded in book values. The adjustment $-\rho_{i,t+1}\,\theta_{i,t+1} + \theta_{i,t}$ accounts for any cash-flow or discount-rate effects reflected in the market return that are omitted from the book return.

To aggregate individual fund returns to index returns, we rely on a second first-order approximation in which the average log return is approximately the log average return (see Appendix C). We cannot consistently estimate $\theta_{i,t}$ for funds that are missing transaction values, but we can consistently estimate the cross-sectional average value of the log return each quarter. Intuitively, the true residual vanishes in the average.

To illustrate, suppose that we observe θ_{it} and a set of characteristics, $\boldsymbol{x_{it}}$, across all funds in each quarter. If we project θ_{it} on a panel of characteristics, $\boldsymbol{x_{it}}$, that includes time fixed effects,

$$\theta_{it} = \boldsymbol{x'_{it}b} + e_{it}, \tag{28}$$

then we can recover the true average value of θ_{it} in each quarter as

$$\bar{\theta}_t = \boldsymbol{\bar{x}_t'b}, \tag{29}$$

for any choice of $\boldsymbol{x_{it}}$, where $\boldsymbol{\bar{x}_t}$ denotes the average value of $\boldsymbol{x_{it}}$ across funds in quarter t. Since we do not observe $\theta_{i,t}$ for all funds, we consistently estimate $\boldsymbol{b}$ under the model specification and assumptions of Heckman (1979). Section III.C below provides additional details. Given a consistent estimate of $\boldsymbol{b}$, define $\hat{r}_{i,t+1}$ as

$$\hat{r}_{i,t+1} = n_{i,t+1} - \rho_{i,t+1} \, \boldsymbol{x}'_{it+1}\boldsymbol{b} + \boldsymbol{x}'_{it}\boldsymbol{b}. \tag{30}$$

While we do not observe θ_{it} for every fund in our index, we *do* observe $n_{i,t+1}$, $\boldsymbol{x}'_{it+1}$, and $\rho_{i,t+1}$ for the full population. Given our consistent estimate of $\boldsymbol{b}$, it then follows that the cross-sectional average value of $\hat{r}_{i,t+1}$ across funds in each quarter is a consistent estimate of $\bar{r}_{t+1}$, the average value of $r_{i,t+1}$ as defined in (26). Since the average log return is the log of the average return to a first-order approximation (Appendix C), average log return therefore represents an estimate of the log return for the index.[13]

C. Two-Step Heckman Approach

To estimate $\boldsymbol{b}$ in (28) we follow the two-step approach of Heckman (1979). In the first step we estimate the parameters of a selection equation,

$$y_{it} = 1[\boldsymbol{z}'_{it}\boldsymbol{c} + u_{it} > 0], \tag{31}$$

where y_{it} is a dummy variable that equals one when fund i transacts at time t, and $\boldsymbol{z}_{it}$ represents a vector of fund characteristics observable across all funds in the index. A consistent estimator of the parameter vector $\boldsymbol{c}$ is available from a first-stage probit estimation of the selection equation using all funds in the portfolio. We then estimate the inverse Mills ratio, $\hat{\lambda}_{it}$, as

$$\hat{\lambda}_{it} = \frac{\phi(\boldsymbol{z}'_{it}\boldsymbol{c})}{\Phi(\boldsymbol{z}'_{it}\boldsymbol{c})}, \tag{32}$$

where $\phi(\cdot)$ is the normal distribution function and $\Phi(\cdot)$ is the cumulative normal distribution function. The second step is to estimate the equation specified in (28) using ordinary least squares (OLS) after including $\hat{\lambda}_{it}$ as an additional explanatory variable,

$$\theta_{it} = \boldsymbol{x}'_{it}\boldsymbol{b} + \hat{\lambda}_{it}d + e_{i,t}. \tag{33}$$

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[13] Note that $\rho_{i,t}\boldsymbol{x}'_{it}\boldsymbol{b} = \rho_{i,t}(\theta_{i,t} - e_{i,t})$ and that the approximation parameter, $\rho_{i,t}$, varies slightly across funds in our application. To ensure that the average value of $\rho_{i,t}e_{it}$ across funds in each quarter is zero, we can appropriately include $\rho_{i,t}$ as one of the characteristics in $\boldsymbol{x}_{it}$. In practice, however, this makes very little difference in results.

The estimate of $\boldsymbol{b}$ in (33) will be consistent under the following four assumptions: (i) $\boldsymbol{z_{it}}$, $\boldsymbol{x_{it}}$, and y_{it} are observed for every fund in the portfolio and θ_{it} is observed whenever $y_{it} = 1$, (ii) e_{it} and $u_{i,t}$ are independent of both $\boldsymbol{z_{it}}$ and $\boldsymbol{x_{it}}$ with zero mean, (iii) $u_{i,t}$ is distributed Normal(0,1), and (iv) $E(e_{i,t}|u_{it}) = \delta u_{it}$, where δ is a constant. See, for example Wooldridge (2010), p. 803. Robust identification that does not rely on assumption (iii) requires an exclusion restriction — a variable in the selection equation that is uncorrelated with the book-to-market ratio. To help ensure that assumption (ii) holds, in practice all explanatory variables in the selection equation, $\boldsymbol{z_{it}}$, are often included as explanatory variables in the primary equation of interest, $\boldsymbol{x_{it}}$.

Section IV provides details on the exact explanatory variables used in our model. We rely on an excluded instrument to achieve robust identification that does not depend on the normality assumption (iii). All other explanatory variables in the selection equation are included in the pricing equation.

Our approach differs somewhat from traditional hedonic techniques to estimate price indices (e.g., Gatzlaff and Haurin (1998) and Hwang, Quigley, and Woodward (2005)). The objective in estimating a traditional hedonic model is to understand price changes for a set of differentiated goods conditional on observed attributes such as quality or specific features. To mirror the typical portfolio definition of an index, we instead define a population of assets and seek to understand changes in the value of the aggregate population for which attributes, such as those related to expected cash flows and discount rates, change over time. An alternative approach to building price indices involves using repeat sales (e.g, Peng (2001)). Repeat sales are too infrequent in our transactions data to consider this approach.

D. Explanatory Variables

The key variables in the pricing equation are time fixed effects, or state variables that represent time fixed effects, to ensure that the mean residual is zero within each quarter across all funds in the population. If funds were selected at random, we would only need to estimate the pricing equation with time fixed effects to consistently estimate the average log book-to-market ratio across our population of funds. As discussed above, because we are estimating the parameters of a sample selection model, we

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include all explanatory variables og the selection equation given in (31) that may be correlated with log book-to-market ratios in the pricing equation (33) to help ensure that the pricing residual is independent of all explanatory variables in the selection equation. (See assumption (ii) above.) Our primary objective is simply to estimate the cross-sectional *average value* of $\theta_{i,t}$ each period, not to forecast the value of $\theta_{i,t}$ for individual funds or to necessarily understand how $\theta_{i,t}$ relates to specific characteristics. In Appendix Table HI we demonstrate that our results are robust to the inclusion of either state variables or time fixed effects, with or without the additional fund-specific characteristics in the pricing equation.

E. Estimating Index Parameters

We estimate index parameters (such as alphas and betas) for our PE index. We compare our estimated parameters against those estimated for other indices, including a Preqin NAV-based index constructed using the same funds that are in our market price index, the Burgiss index, which is also a NAV-based index, and the S&P Listed Private Equity Index.

So far we have described a process to estimate the quarterly log return for our index. It is well known that the intercept in a regression of excess log returns on excess log market returns cannot be interpreted as alpha. A common solution is to estimate the intercept using annualized log returns, and then adjust under the assumption that log returns are normally distributed (see, for example, Cochrane (2004) and Axelson, Sorenson, and Stromberg (2014)). To estimate parameters for the Preqin, Burgiss, and S&P indices mentioned above we use simple quarterly returns. To follow suit, we exponentiate our estimate of the quarterly index log return, the cross-sectional average value of $\hat{r}_{i,t+1}$ given in (26), to arrive at a consistent estimate of the simple return for our index before calculating index parameters. We obtain similar results if we run regressions using excess log returns and adjust the intercept based on the normal assumption as others have done. These results are given in Panel A of Appendix Table HI.

The transactions in the PE secondary market upon which we build our index are highly nonsynchronous, occurring at various times throughout the quarter. Standard measures of alpha, beta, and volatility using nonsynchronous data are in general inconsistent. We therefore follow Dimson (1979) to

estimate index betas by regressing returns on contemporaneous and lagged values of the market return, for an appropriate number of lags, and summing the slope coefficients. We estimate alpha as

$$\alpha = \left(\bar{r}_{p,t+1} - \bar{r}_{f,t+1}\right) - \beta\left(\bar{r}_{m,t+1} - \bar{r}_{f,t+1}\right), \tag{34}$$

where $\bar{r}_{p,t+1}$ represents our average estimated index return across quarters, $\bar{r}_{f,t+1}$ is the average return on short-term T-bills, $\bar{r}_{m,t+1}$ is the average public market return, and β represents our Dimson adjusted beta. We obtain consistent estimates of volatility, Sharpe ratios, and correlation under the model of Dimson (1979) as outlined in Appendix D.

IV. Data on Transactions in the Private Equity Secondary Market

In this section we briefly describe the data and variables used to estimate the index. Our transaction data come from a large intermediary in the PE secondary market, extending the data used by Nadauld et al. (2019). These data identify the fund that is sold, the total capital committed by the seller, the amount unfunded by the seller, the purchase price, and the transaction date for all transactions consummated through this intermediary from January 2006 through December 2018. We clean the data as detailed in Appendix E and pull the most recent transaction for each fund each calendar quarter for funds with a total commitment greater than $500 million.

We obtain data on other fund characteristics, such as calls, distributions, NAV, fund LP type, and size, from Preqin, for the period January 2006 through June 2018, and clean these data as detailed in Appendix E. The Preqin data come from a variety of sources including public filings and reports, submissions by GPs, and Freedom of Information Act (FOIA) requests. While there are other data sets that may be more representative of the entire fund universe, Preqin is the only database that provides fund names, thus making it possible to merge in our secondary market transaction data.

Within each calendar quarter we sum all contributions and distributions for a given fund. We then merge transactions data with the Preqin sample, some of which is done by hand. Again, details can be found in Appendix E. After this process, we end up with a sample of 596 funds for which we have fund-level

data. We observe 839 transactions on 287 of these funds between the first quarter of 2006 and the second quarter of 2018.

We propose using eight explanatory variables in our selection equation: the log size of each fund, two age dummies, fund PME, the fraction of LPs for each fund that are pension funds, and three state variables that proxy for quarter fixed effects. For our main results we use state variables rather than quarter fixed effects to conserve power and because there is little economic insight to be obtained from the coefficients on the fixed effects themselves. We demonstrate that our results are robust to this choice by replacing the three state variables with time fixed effects. We also illustrate that our results are robust to whether or not we include the additional explanatory variables from the selection equation in the pricing equation along with state variables or time fixed effects. We report these results in Appendix Table HI.[14]

We measure size as total commitments by LPs to the fund. The first age dummy equals one if the fund age (calculated in years relative to the vintage year) is less than four years, and zero otherwise. The second age dummy equals one if the fund age is greater than nine years, and zero otherwise. We also use the Kaplan-Schoar (2005) PME for each fund-quarter, measured using cash flows from Preqin and using NAV at the end of the quarter as the terminal value. The fraction of LPs for each fund that are pension funds comes from Preqin.

Fund size and age are likely to be associated with asymmetric information about the fund or GP, and therefore the likelihood that a deal agreeable to both parties can be reached. Fund age is also directly related to the period over which fixed transaction costs can be amortized, suggesting that younger funds may be less likely to transact. PME captures information about the performance of funds that may be associated with supply and demand. Pension fund objectives are likely to differ from those of other investors, making PE funds with high pension ownership less likely to be sold. We further discuss pension fund ownership and objectives below.

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[14]In the context of the sample selection model, the inclusion of time fixed effects prevents any variable that changes over time, such as a decrease in liquidity, from adversely affecting model estimates.

The three state variables we use include the log-value-weighted book-to-market ratio for small cap stocks, the TED spread, and a measure of total assets under management for the PE industry. We measure the small-cap book-to-market ratio using stocks with share code 10 and 11 in data from the Center for Research in Security Prices (CRSP) with less than $500 million in market cap. We obtain the TED spread, calculated as the spread between three-month LIBOR based on U.S. dollars and three-month Treasury yields, from the St. Louis Federal Reserve. We measure these two state variables at the end of the month prior to the fund's transaction date for funds that transact, and at the end of the second month of each quarter for funds that do not. Finally, we measure total assets under management, obtained from Bloomberg, at the end of the prior quarter and scale by the total number of firms in the U.S. with between 20 and 500 employees, obtained from the U.S. Census, measured at the end of the prior year.

We use the fraction of LPs invested in a fund that are pension funds, PF_{it}, as an instrument. The investment objectives of pension funds differ from those of other investors: pension funds manage investment cash flows to match the timing of their liabilities but face unique regulatory incentives to refrain from dynamically hedging short-term liability risks (van Binsbergen and Brandt (2015)). As such, pension funds may be less likely to engage in PE secondary markets than other investors, such as a fund-of-funds. Consistent with this idea, Nadauld et al. (2019) document that empirically, pension funds sell their PE stakes much less frequently than other investors.

To be a valid instrument, $PF_{i,t}$ must be uncorrelated with log book-to-market after controlling for other observables. Book-to-market ratios are well known to predict stock returns, suggesting that they proxy for risk or mispricing in the cross section. Evidence on the relation between pension ownership and mispricing is somewhat mixed. Lerner, Schoar, and Wongsunwai (2007) document that pension funds outperform other LPs (other than endowments) in their PE reinvestment decisions, but their result does not

Electronic copy available at: https://ssrn.com/abstract=4133946

replicate out of their 1990s sample, which does not overlap with our sample period (see Sensoy, Wang, and Weisbach (2014)).[15]

Table I reports summary statistics. For this table we separate records with transactions from those without. Consistent with prior findings, funds on average transact at a discount relative to NAV (see Nadauld et al. (2019)). The overall average log book-to-market ratio is 0.19, corresponding roughly to a 17% discount $(1/\exp(0.19) = 0.83)$, and the median is 0.10, corresponding to a 10% discount.

< INSERT TABLE I HERE >

The deviation between a fund's NAV and its market price will depend in large part on how the market discounts future expected cash flows relative to the GP. Hence, a trade for less than NAV is not necessarily reflective of a liquidity discount. The economic discount or premium at which a transaction occurs should be measured relative to the (unobservable) underlying value of the fund's assets, not the NAV (see Nadauld et al. (2019)).

Funds that transact in the secondary market tend to be larger and older than average. The average log fund size for funds that transact is about 21.95 (corresponding roughly to a size of $3.4 billion) compared to an average log fund size for funds that do not transact of about 21.28 (corresponding roughly to a size of $1.7 billion). The average age is 8.6 years for all transacting funds, and 6.4 years for non-transacting funds. In addition, funds that transact display slightly higher average PMEs in the range of 1.15 compared to 1.13 for non-transacting funds. For the average fund in our sample that transacts, about 53% of the LPs are pension funds, as indicated by the summary statistics on PF_{it}, compared with 56% for the average fund that does not.

Figure 1 shows the number of transactions per quarter for the full sample; in total there are 839 transactions. The figure highlights the rapid growth in the secondary market: the years 2017 and 2018 record almost five times the number of transactions as the years 2006 and 2007.

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[15] Others find that pension funds make poor investment decisions relative to other institutions (see Hochberg and Rauh (2012), and Andonov, Hochberg, and Rauh (2018)).

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< INSERT FIGURE 1 HERE >

V. Estimates of Secondary Market-Based Private Equity Indices

A. Selection and Pricing Equations

Table II reports estimates of the parameters for the selection and pricing equations specified in (31) and (33), respectively. Columns (1) through (4) report results for the Heckman model, while columns (5) and (6) report basic OLS estimates of the pricing equation without the inverse Mills ratio. We estimate standard errors for the Heckman model using a panel bootstrap, which accounts for the inverse Mills ratio being a generated regressor, and cluster by time. We estimate White (1980) standard errors for the OLS model and cluster by time.

< INSERT TABLE II HERE >

The estimates presented in columns (1) and (2) suggest that a number of variates are associated with fund selection. First, our proposed pension fund instrument does predict selection into the transaction sample even after controlling for other variables. Pension funds appear to be substantially less likely to sell than their institutional investor counterparts. In addition, funds are more likely to transact in quarters with a higher TED spread and in quarters with higher total industry assets under management. Among the fund-specific variables, larger and older funds are more likely to transact than smaller and younger funds, consistent with the idea that larger and older funds may be associated with less asymmetric information or lower relative transaction costs. Four- to nine-year-old fund transactions are the most common age type in our data in terms of absolute number but the percentage of four- to nine-year-old funds that transact is smaller than the percentage of older funds that transact. Table II also indicates that transacting funds tend to have lower average PMEs at the time of the transaction after controlling for other variates. This finding suggests that sellers tend to bring lower performing funds to market, all else equal.

Columns (3) and (4) of Table II report Heckman (1979) estimates of the parameters for the pricing equation given in (33), with the log-book-to-market ratio, θ_{it}, as the dependent variable. A Campbell-Shiller (1988) decomposition indicates that higher values of θ_{it} is associated with either higher market

discount rates, lower book discount rates, or higher future book-to-market ratios. We illustrate this decomposition and show empirically that all variation in θ_{it} is associated with variation in long run market discount rates in Section 7.1.

Column (3) of Table II indicates that θ_{it} tends to be higher when the log book-to-market ratio for public equities is high, with the coefficient on $\log(BM_t)$ equaling 0.79 *(t*-statistic = 3.8). The estimates imply that a one-standard-deviation shock to $\log(BM_t)$ leads to an expected change in θ_{it} of 0.18, approximately one-third of the standard deviation of θ_{it}. Our index tends to have a higher future expected return when the public book-to-market ratio is high, that is, when prices for public equities are relatively low. Another important explanatory variable in the pricing equation is overall industry assets under management ($IAUM_t$). The coefficient on $IAUM_t$ is -0.20 with a *t*-statistic of -2.4. A one-standard-deviation shock to $IAUM_t$ is associated in a change in θ_{it} of -0.19, again approximately one-third of the standard deviation of θ_{it}. Future expected returns for our index, as indicated by a higher book-to-market ratio, tend to drop with overall industry assets under management. This finding is similar to that of Pastor, Stambaugh, and Taylor (2015), who find empirical evidence for decreasing returns to scale in active mutual fund management at the industry level.

The coefficient on the inverse Mills ratio in the pricing equation is not significantly different from zero, with a point estimate of 0.06 and a *t*-statistic of 0.3. After controlling for our chosen economy-wide and firm-specific characteristics, innovations in price and selection appear to be uncorrelated. In this case, even OLS estimates of the pricing equation are consistent. In fact, the OLS estimates of the pricing equation (columns (5) and (6)) are very close to those of the Heckman estimates of the pricing equation.

B. Private Equity Indices over Time

We create two equally weighted market PE indices, one using funds of all ages, and a second using only funds that are four to nine years old. Figure 2 graphs our PE indices over the 2006 to 2018 sample period on a log scale. For comparison we also present the performance of the public equity market (from Ken French's website), two equally weighted indices based on Preqin reported NAVs (one using funds of

all ages and another using only funds that are four to nine years old), the Burgiss index, and the S&P Listed Private Equity Index. We create the Preqin indices using the same population of funds that we use to create the market-based PE indices and measure the return as the cross-sectional average of $\exp(n_{i,t+1}) - 1$, where $n_{i,t+1}$ is the log return defined in (19). Our Preqin indices are quite similar to the Burgiss index. The correlation between our Preqin index (all ages) and the Burgiss index is 0.97, while the correlation between our Preqin index (four to nine years) and the Burgiss index is 0.95.

Figure 2 illustrates that the two market PE indices are more volatile than the three NAV-based indices (Preqin (all ages), Preqin (four to nine years), and Burgiss). The well-known excessive "smoothness" of NAVs is generally attributed to the use of stale information by GPs. It is also possible that NAVs may not fully reflect variation in market discount rates. We further explore this issue in Section VI.A. The only sharp decline in equity markets during our sample period occurred during the 2008 financial crisis. During this period, the NAV indices declined somewhat because assets were written down, but did not decline nearly as much as public equity markets. In contrast, the decline in the market PE indices over 2008 is similar to that in public equity markets. Over 2008 the public equity market index declined 37%, the S&P Listed Private Equity Index declined 64%, while the market PE index (all ages) declined 60%. Our Preqin NAV-based index (all ages), in contrast, fell by only 25%, and the Burgiss NAV-based index dropped by only 27% during 2008. Even though PE NAVs were written down by nearly one-quarter of their value during the 2008 crisis, their actual value most likely declined similar to public equity markets at that time.

< INSERT FIGURE 2 HERE >

C. Risk and Return of Private Equity Indices

In this section we report estimates of expected returns, beta, alpha, volatility, and Sharpe ratios for market-based indices of buyout funds. To implement the Dimson (1979) adjustment, we report regressions of index returns on lagged market returns in Table III. The first four columns report estimates for the two market-based PE indices, while the remaining columns contain estimates for the S&P Listed Private Equity

Index, Preqin, and Burgiss indices. We compute standard errors by GMM using the Newey-West (1987) estimate of the spectral density matrix.[16]

< INSERT TABLE III HERE >

Each of the NAV-based indices loads significantly on lagged market returns through three lags. The S&P Listed Private Equity Index appears to be uncorrelated with lagged market returns. We find that the market-based PE indices load significantly on lagged market returns through only one lag. The estimates in Table III suggest that when applying adjustments based on Dimson (1979), we should account for cross-autocorrelation with market returns at one lag for the market-based PE index, at three lags for the NAV-based indices, and zero lags for the S&P Listed Private Equity Index.

Table IV reports index parameter estimates for the various indices including the market-based PE indices. For this table we compute GMM standard errors using the Newey-West (1987) spectral density matrix along with the delta method, as needed. Consistent with Figure 2, the estimates in Table IV indicate that buyout funds have performed well over our sample period. The average return for the market-based index using funds of all ages is 14% and using funds that are four to nine years old is 22%. In contrast, NAV-based index returns averaged from 11%-14%, while the S&P Listed PE Index averaged a return of only 9% over our sample period.

< INSERT TABLE IV HERE >

Our market-based index using funds of all ages is 1.79, and for four- to nine-year-old funds is 1.76. These estimates are larger than most betas reported in prior literature. As discussed above, betas estimated using cash-flow data are unlikely to pick up variation in PE discount rates that manifests in actual market prices for PE. To understand the magnitude of our estimate, recall that PE funds are portfolios of equity positions in leveraged buyouts. Since buyouts tend to be much more highly levered than public firms,

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[16] The market-based PE indices contain measurement error that is reasonably independent of the actual index return. Since this measurement is reflected in the dependent variables of our regressions, it is absorbed in the regression residuals.

Modigliani-Miller Proposition 2 implies that buyouts should have substantially higher betas than public firms. For example, Axelson et al. (2013) report a mean debt-to-total-capital ratio of 70% in their sample of 1,157 leveraged buyouts (LBOs), with mean leverage closer to 50% during our 2006 to 2018 sample period. In contrast, typical large publicly traded firms have a debt-to-total-capital ratio of approximately 20% to 25%. If the firms experiencing buyouts have asset betas equal to one, which reflect *both* discount-rate and cash-flow effects, and debt betas are positive but relatively small, the equity portion of the LBO should nonetheless have a beta close to 1.60 to 1.80, which is consistent with our estimates.[17]

For the market PE index using funds of all ages, we document an alpha of -2% annually. The vast majority of the literature finds that buyout funds outperform after accounting for risk manifest in PE cash flows. Our alpha accounts for both the covariance of cash flows with the SDF and the covariance of discount rates with the SDF. The alpha of the market PE index using four- to nine-year-old funds is estimated to be relatively high at 6% annualized (though insignificant). It is possible that funds may tend to perform better as they acquire assets, though we cannot reject the null that the alpha for four- to nine-year-old funds is zero.

Using log returns we find betas of 1.95 using funds of all ages and 1.86 using four- to nine-year-old funds. The adjusted alphas under the normal assumption for both indices are both very close to zero. We report these results together with those of other robustness tests in Appendix Table HI.

The S&P Listed Private Equity Index produces a beta of 1.74, which is similar to that of our market-based PE indices. Average returns and alphas, however, are quite different. The S&P Listed PE index earned an average return of 9% (*t*-statistic = 1.0) and an alpha of -7% over our sample period (*t*-statistic = -2.7).

Relative to other indices in Table IV, the estimated volatilities and betas for the NAV indices are quite low. These results suggest that NAV-based indices, even after Dimson adjusting, fail to capture important dynamics in PE that is related to public equity markets, such as variation in discount rates. We further explore this issue in Section VI.A. Adding more lags to the Dimson adjustment does *not* cause the

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[17] See Axelson, Sorensen, and Strömberg (2014) for more discussion.

betas of NAV-based indices to become more similar to those of the transaction-based indices or the S&P Listed Private Equity Index.[18] In addition, there is a large difference in volatility between the market-based PE indices (about 34%) and the NAV-based indices (ranging from 14% to 20%). The volatility of the S&P Listed Private Equity Index (30%) is also larger than that of the NAV-based indices. In Section VI.A we provide evidence that NAV-based indices do not fully account for variation in market PE discount rates, which represents another cause for NAV smoothness above and beyond stale information.

VI. Discount Rate Risk in Private Equity

In this section we explore two questions. First, how much do PE discount rates vary over time? Second, to what degree does the discount rate premium identified in Section II.B and equation (17) empirically create a wedge between GPME and alpha? To answer the first question, we run regressions of long-run PE secondary market returns on log book-to-market ratios motivated by the Campbell-Shiller (1988) present value identity. To answer the second question, we empirically estimate and compare unconditional GPME for the funds in our index with the unconditional SDF alpha of the index itself. We then also estimate and compare these two performance measures for a set of synthetic funds that invest in US public equities.

A. Variation in Book-to-Market Ratios and Discount Rates

In this section we explore how much PE discount rates vary over time. Equation (26) shows that the difference in log market returns and log book returns can be written as a linear function of book-to-market ratios to a first-order approximation (Vuolanteenaho (2002)). Taking a cross-sectional average of the relation specified in (26) across funds, we obtain

$$\frac{1}{N}\sum_{i=1}^{N} n_{i,t+1} - r_{i,t+1} = \frac{1}{N}\sum_{i=1}^{N} \rho_{i,t+1}\, \theta_{i,t+1} - \theta_t, \tag{35}$$

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[18] Even if we include seven lags in the Dimson adjustment as suggested by Metrick and Yasuda (2010), we find that NAV-based betas over our sample period are only 0.73 to 0.85.

which can be written as

$$\bar{n}_{t+1} - \bar{r}_{t+1} = \bar{\rho}_{t+1}\,\bar{\theta}_{t+1} - \bar{\theta}_t + \bar{c}_{\rho\theta,t+1}, \tag{36}$$

where $\bar{c}_{\rho\theta,t+1}$ denotes the empirical covariance between $\rho_{i,t+1}$ and $\theta_{i,t+1}$ in the cross section across funds,

As explained in Section III.B and Appendix B, the approximation parameter, $\rho_{i,t+1}$, can take on only one of three values, all close to 1.0. We empirically find time-series variation in $\bar{\rho}_{t+1}$ to be trivially small, with a quarterly standard deviation of about 0.01 over our sample. Further, the mean value of $\bar{\rho}_{t+1}$ over time is statistically indistinguishable from 1.0. We therefore motivate the empirical exercise of this section by setting $\bar{\rho}_{t+1} = 1$ in each quarter.

Iterating on (36) through time $t + k$ after setting $\bar{\rho}_{t+1} = 1$ and taking expectations conditional on information at time t, we obtain

$$\bar{\theta}_t = \sum_{j=1}^{k} E_t[\bar{r}_{t+j}] - \sum_{j=1}^{k} E_t[\bar{n}_{t+j}] + E_t[\varepsilon_{t+k}], \tag{37}$$

where ε_{t+k} is given by

$$\varepsilon_{t+k} = \sum_{j=1}^{k} \bar{c}_{\rho\theta,t+j} + \bar{\theta}_{t+k}. \tag{38}$$

Equation (37) says that variation in the average log book-to-market ratio, $\bar{\theta}_t$, must be associated with variation in market PE discount rates, $E_t[\bar{r}_{t+j}]$, variation in book discount rates used to compute NAV, $E_t[\bar{n}_{t+j}]$, or variation in $E_t[\varepsilon_{t+k}]$. We can better understand this variation by regressing long-run market returns, long-run book returns, and long-run values of ε_{t+k} on $\bar{\theta}_t$ in three separate regressions.

$$\sum_{j=1}^{k} \bar{r}_{t+j} = a_r + \beta_r \bar{\theta}_t + w_{r,t+k}$$

$$-\sum_{j=1}^{k} \bar{n}_{t+j} = a_n + \beta_n \bar{\theta}_t + w_{n,t+k} \tag{39}$$

$$\varepsilon_{t+k} = a_\varepsilon + \beta_\varepsilon \bar{\theta}_t + w_{\varepsilon,t+k}.$$

Given the relation provided in (37), it follows that

$$1 = \beta_r + \beta_n + \beta_\varepsilon. \tag{40}$$

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We can interpret the slope coefficients β_r, β_n, and β_ε as the fraction of variation in $\bar{\theta}_t$ associated with its ability to predict future PE returns, book returns, and ε_{t+k} (Cochrane (2011)).

In Table V we report results for the slope coefficients of the three regressions given in (39). Given the evidence in Table III that lagged returns predict future returns due to nonsynchronous trading and stale NAVs, we estimate these regressions with and without controls for lagged transaction and book returns. Including lagged transaction returns in particular on the right side of these regression may help control for elements of $\bar{\theta}_t$ that overlap with $\bar{r}_{t+1}$ from nonsynchronous prices. On the left side of each panel we report results using $\bar{\theta}_t$ as the sole explanatory variable. On the right side we report results that also control for $\bar{r}_t$ and $\bar{n}_t$ as explanatory variables. In all panels the dependent variable is either $\bar{r}_{t+j}$, $\bar{n}_{t+j}$, or ε_{t+k}. The data is quarterly. Panel A reports results for $k = 1$ (one quarter). Panels B and C report results for overlapping regressions where $k = 4$ (one year) and $k = 20$ (five years). We compute Hansen-Hodrick (1980) standard errors that account for the overlapping returns in our regressions, taking the estimated book-to-market ratios from our index as given.

<center>< INSERT TABLE V HERE ></center>

On the left side of Panel A we see that at the quarterly horizon, about 89% of the variation in book-to-market ratios is associated with its ability to predict ε_{t+k}. The value of β_ε on the left side of Panel A is estimated to be 0.89 and highly significant, while the regression R^2 is 0.75. These results are driven by persistence in book-to-market ratios. Regressing $\bar{c}_{\rho\theta,t+1}$ alone on $\bar{\theta}_t$ produces an insignificant slope coefficient of 0.0002. In addition, while the estimate of β_r is small and insignificant on the left side of Panel A, the estimate jumps to 0.31 and becomes significant after controlling for lagged returns on the right side of Panel A.

In Panel B of Table V we see that at longer horizons, more variation in book-to-market ratios is explained by its ability to predict returns. In the regression of annual returns on book-to-market ratios one year prior, the estimated value of β_r is 0.45 but still insignificant. After controlling for lagged returns the estimate of β_r increases to 0.65, though it is still insignificant. Moreover, most of the variation in book to-

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market is associated with its ability to predict ε_{t+k}. The estimated value of β_ε on the left side of Panel B is 0.83, while it is 0.56 after controlling for lagged returns on the right side of Panel B.

In Panel C of Table V, the estimated value of β_r for the regressions of five-year returns on book-to-market ratios five years prior is 1.65 and highly significant, while the R^2 is 54%. In contrast, estimates of β_n and β_ε are insignificant. In fact, estimates of β_n are insignificant in *all* panels, and in Panel C the point estimate of β_n is negative. The negative coefficient on β_n is consistent with the view that book discount rates respond weakly to and are positively correlated with market discount rates.[19] Controlling for lagged returns makes little difference in the long-run regressions of Panel C. As might be expected, quarterly returns have little power to predict five-year returns even when prices are non-synchronous throughout each quarter. As such, controlling for lagged transaction and book returns in these regressions has little effect on the results.

Expected long-run PE returns in (39) are given by $\beta_r\bar{\theta}_t$, and an estimate of the variation in long-run expected PE returns is given by $\beta_r^2 Var(\bar{\theta}_t)$. We estimate $Var(\bar{\theta}_t)$ to be 0.05 (using annual data), implying that the standard deviation of five-year expected PE returns is 0.37. Cochrane (2011) estimates the standard deviation of five-year expected returns for the public market portfolio using annual data to be 0.29. The beta of our index, however, is 1.79, whereas Cochrane's results are for a unit-beta portfolio. Hence, a more appropriate comparison from public markets is the standard deviation of five-year expected returns for a levered market portfolio with a beta of 1.79, that is, $1.79 \times 0.29 = 0.52$. From this perspective, variation in PE discount rates may be somewhat *less* than variation in public market discount rates. Variation in risk aversion and sentiment of the marginal investor in PE may be somewhat muted

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[19] Consider a regression of negative book returns on market returns, $-\sum_{j=1}^{k}\bar{n}_{t+j} = \gamma_0 + \gamma_1\sum_{j=1}^{k}\bar{r}_{t+j} + z_{t+k}$. Then $Cov\left(-\sum_{j=1}^{k}\bar{n}_{t+j},\bar{\theta}_t\right) = \gamma_1 Cov\left(\sum_{j=1}^{k}\bar{r}_{t+j},\bar{\theta}_t\right) + Cov(z_{t+k},\bar{\theta}_t)$ and $\beta_n = \gamma_1\beta_r + \beta_z$, where β_z is the slope coefficient in a regression of z_{t+k} on $\bar{\theta}_t$. In this case, β_n may be less than zero if γ_1 is sufficiently negative (implying a positive relationship between book and market returns) and β_z is sufficiently small. For five-year returns we estimate γ_1 to be -0.39 and β_z to be 0.20. A similar explanation illustrates why β_r is above 1.0.

relative to that of the marginal investor in public equity. In contrast, the standard deviation of book discount rates is only 0.10. NAVs are too smooth from the perspective of an investor with access to secondary markets, not only because they reflect stale information, but also because they fail to reflect variation in market discount rates for PE.

B. GPME and Alpha: Real and Simulated Funds

Equation (17) illustrates the relationship between unconditional GPME and expected conditional alpha. In this section we report empirical estimates of unconditional GPME using funds in our index and the unconditional SDF alpha of the index itself. To further investigate the relation between GPME and alpha as empirical measures of performance, we create a set of synthetic PE funds that invest in U.S. public equities and compare estimates of GPME and alpha for these funds.

Our approach to estimate GPME and conduct inference is identical to that in Korteweg and Nagel (2016),[20] and thus here we highlight only a few details. The SDF we use may be interpreted as the CAPM SDF that arises within a representative-agent economy under the assumption of constant relative risk-aversion (Giovannini and Weil (1989)),

$$M_{t:t+j} = \exp(a_0 j + a_1 r_{m,t:t+j}),\tag{41}$$

where $r_{m,t:t+j}$ is the log market return from t to $t + j$, and a_0 and a_1 are parameters chosen to price two benchmark funds created for each PE fund in the sample. The two benchmark funds for PE fund i make capital calls and distributions to mimic the timing of flows to PE fund i following a specified algorithm, where one benchmark fund invests all capital in a publicly traded market index fund and the other invests all capital in short-term T-bills. Parameters a_0 and a_1 are chosen such that the average realized GPMEs for the two benchmark funds across all PE funds in the sample are both identical to zero. To conduct inference, we compute a GMM J-statistic to test the null that GPME pricing errors across all funds are zero.

We build our market PE index using funds with recorded cash flows in Preqin from 2006 to 2018. The vintages of these funds span from 1988 to 2018. We scale all cash flows to a $1 commitment and use

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[20] We thank these authors for supplying us with Matlab code.

NAV as a terminal value for funds not yet liquidated by the end of our sample. We find the GPME for funds in our index to be 0.26 with a *J*-statistic *p*-value of 0.31. Korteweg and Nagel (2016) report the GPME for venture capital funds of pre-1998 vintage to be 0.423 and of post-1998 vintage to be 0.048. Our point estimate of the GPME for buyout funds in our index is about two-thirds that of venture funds pre-1998, a period of strong performance for venture funds. Although economically meaningful, the point estimate is not significantly different from zero, which highlights the general difficulty of precisely measuring GPME with short samples and overlapping cash flows. We also find the average annualized IRR for funds in our index to be 13.8%.

Using the same SDF that we use to estimate GPME given in (41), we also measure the SDF alpha of our index as

$$\alpha = \bar{R}_t - \frac{1}{\bar{M}_{t:t+1}} + \frac{Var(M_{t:t+1})}{\bar{M}_{t:t+1}} \beta_M, \tag{42}$$

where R_t is the gross index return and β_m represents a Dimson-adjusted beta of R_{t+1} relative to $M_{t:t+1}$.[21]

To measure the *t*-statistic for our estimate, we first estimate the GMM-Newey-West covariance matrix of the components of (42) taking the SDF parameters as given, and then estimate the standard error for α using the delta method. For our index, we estimate the SDF alpha to be 5.4% on an annualized basis with a *t*-statistic of 0.40. The annualized unconditional CAPM alpha for our market-based index is -2% (reported in column (1) of Table IV) and is also statistically insignificant. While the GPME is economically meaningful, the SDF and CAPM alphas are relatively less impressive.

Given that unconditional alpha is zero, equation (17) illustrates that the wedge between GPME and alpha emerges from a discount-rate risk premium that lowers alpha relative to GPME. While our empirical results suggest that such a wedge from a positive discount-rate risk premium exists among the PE funds we study, we lack power to estimate GPME with much precision using the overlapping cash flows of the funds in our universe. Statistically, we cannot reject that GPME itself is zero. In addition, our results may be

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[21] We use one lag in the Dimson adjustment given the evidence of columns (1) and (2) of Table III.

driven by specific features of PE secondary markets, transaction costs, or the manner in which we construct our PE secondary market indices. To provide further evidence that discount-rate risk separates cash-flow-based measures of performance from standard measures, we estimate GPME and alpha using a group of investments in public companies for which valuations and portfolio construction are transparent, and where transaction costs are minimal.

We sort stocks with share code 10 and 11 from the CRSP universe into size decile portfolios and create the cash flows for a series of artificial PE funds that make capital calls and invest all capital in an assigned size decile portfolio from 1980 to 2018. To create the cash flows for these artificial funds, we adopt the same mimicking algorithm of Korteweg and Nagel (2016) to create the benchmark funds necessary to estimate GPME. The timing of cash flows for our artificial funds mimics those of a "representative fund" that we create by averaging cash flows across all PE funds in our sample after scaling all flows to a $1 commitment and aligning them in fund-inception time. Cash flows for this representative fund are given in Figure 3. For each decile, a new fund begins with a capital call every six months, invests all capital in the assigned decile portfolio, makes subsequent calls and distributions to mimic the timing of cash flows of the representative fund, and liquidates after 10 years. We relegate further details on the creation of our artificial funds to Appendix F. The end result is a series of overlapping cash flows for 590 artificial funds, 59 funds for each decile portfolio. We then estimate the unconditional GPME using the cash flows of funds for each decile and compare this estimate with the estimated SDF and CAPM alphas for the same decile portfolio over the same period. To estimate the SDF alpha and its corresponding *t*-statistic, we follow the same approach that we use to estimate the SDF alpha and corresponding *t*-statistic for our index.

< INSERT FIGURE 3 HERE >

Table VI reports results for the artificial funds. Column (1) reports the decile in which the funds invest capital. In column (2) we report the average equity market cap of firms for each decile in 2020 dollars. Decile-1 funds purchase public firms with an average market cap of $17 million, while decile-10

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funds purchase public firms with an average market cap of $87 billion. Pitchbook reports the median buyout deal size in 2012, the midpoint of our sample, to be about $80 million and in 2018, the end of our sample, to be about $150 million (Pitchbook (2022)). Assuming a 25% debt-to-total-capital ratio, decile-3 firms, with an average market cap of $81.6 million, have total firm value of about $106.8 million and are roughly comparable to the median buyout deal size of funds over our sample as reported by Pitchbook. At the bottom of column (2) we report that the average number of firms per decile over time is 538.

In column (3) of Table VI we report the average IRR for each set of artificial funds. The average IRR for decile-3 is 10%, a little lower than the 13.8% average IRR we estimate for funds in our index. The artificial funds that invest in raw public equities, however, are likely to entail less leverage than funds in our index. For example, Axelson et al. (2013) report a mean debt-to-total-capital ratio of 70% in their sample of 1,157 LBOs, with mean leverage closer to 50% during our 2006 to 2018 sample period. In contrast, typical large publicly traded firms have debt-to-total-capital ratios of approximately 20% to 25%. Greater leverage would increase the IRRs of our artificial funds to some degree.

< INSERT TABLE VI HERE >

In column (4) of Table VI we report the estimated GPMEs for funds that invest in each of the size decile portfolios and in column (5) we report the p-value of the GMM J-statistic. All GPMEs are statistically significant, and all are positive except funds that invest in the largest decile. GPME is estimated with greater precision for our artificial funds than for funds in our index because the cash flows for our artificial funds span a longer period and exhibit less overlap. GPMEs of funds that invest in smaller companies are especially large. The GPME for decile-3 funds is estimated at 0.193 while the GPME for funds in our index that entail somewhat greater leverage is estimated to be a little higher at 0.26, as reported above.

Column (6) of Table VI reports the SDF alphas for each decile portfolio, and column (7) reports the p-value associated with the GMM t-statistic. We report p-values to be consistent with column (5). In contrast to GPME, SDF alphas are all statistically insignificant and tend to be especially low for funds that invest in small-cap stocks. The SDF alpha for decile-3 funds is -0.009 with a p-value of 0.91. Additional

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leverage would only make this *p*-value more negative. Hence, columns (4) and (6) provide compelling evidence that a positive discount-rate risk premium exists, as defined in equation (17) using the SDF given in (41), for firms that are about the same size as companies purchased by funds in our index. Given that the SDF alpha is zero, equation (17) shows that GPME can only be positive if the discount-rate risk premium is positive. This positive discount-rate risk premium lowers alpha relative to GPME.

Column (8) reports the standard CAPM alpha for each decile portfolio, and column (9) reports the *p*-value associated with the GMM-Newey-West *t*-statistic. Here we see that all CAPM alphas are insignificant and close to zero.

In summary, our empirical results suggest that GPME does not appropriately represent the marginal utility gain from an incremental investment in PE for a myopic one-period investor that has access to PE secondary markets and currently holds positions in a stock market fund and risk-free bonds. Transitory price fluctuations that arise from variation in market discount rates can generate a meaningful distinction between GPME and CAPM alpha.

Given the CAPM SDF of Korteweg and Nagel (2016) that we use in our empirical work, our results do not speak to the dynamic portfolio selection problem in PE. We believe this can be an interesting avenue for future research. The expected utility of an unconstrained dynamic LP with secondary market access must be at least as high as the expected utility of a constrained buy-and-hold LP, since the buy-and-hold strategy is among the possible dynamic strategies the unconstrained investor can pursue. Given that some future PE cash flows are negative, however, it may be optimal for some unconstrained LPs to preserve the option to acquire a stake in a PE deal at potentially favorable secondary prices while avoiding high capital calls that potentially arrive in bad states of the world, rather than locking into PE contracts directly with GPs. While expected utility is always higher for investors that face fewer constraints and costs, the marginal utility gain from an incremental investment in PE may at times be lower for an unconstrained dynamic LP than for a buy-and-hold LP. Among other things, to better understand the optimal dynamic portfolio allocation problem, we need to better understand the constraints and costs that prevent LPs from

more fully engaging in PE secondary markets. These may include reputational costs with GPs who may not like having their PE stakes sold in secondary markets and may shut LPs out from future deals.

VIII. Conclusion

Historically, measuring the performance of PE investments has only been possible using cash-flow data. In recent years, however, a secondary market has developed in which investors in PE funds can trade their stakes. Prices from this market provide a source of data useful for measuring the risk and return of PE funds in a similar manner to that commonly used to measure returns for other securities.

We construct indices of buyout performance using a proprietary database of secondary market prices of PE stakes between 2006 and 2018 while carefully accounting for sample selection. Analysis of these indices indicates that discount rates in PE vary considerably, an insight that is not readily available from cash-flow-based measures of PE performance. We confirm this result in multiple ways. First, we derive the theoretical relation between the unconditional GPME of Korteweg and Nagel (2016) and alpha when both are measured relative to the same SDF. We show that alpha accounts for a discount-rate risk premium that creates a wedge between the two measures. We empirically find that our market-based indices of buyout funds track public equity much more closely than the NAV-based indices we consider. The buyout indices we construct have market betas of about 1.79, consistent with the notion that buyouts with increased leverage have higher betas, in marked contrast to the NAV-based indices, whose Dimson-adjusted betas are estimated to be around 0.8. Using our indices we show in simple Campbell-Shiller regressions that discount rates for PE are much more variable than book discount rates, and perhaps somewhat less variable than discount rates for public equity. Even after using standard approaches to adjust for known staleness, NAVs still appear too smooth from the perspective of an investor with access to secondary markets because they fail to reflect variation in market discount rates for PE. Our indices produce CAPM alpha to be close to zero and insignificant, while GPME for funds in the index is economically large. We also show for a sample of synthetic funds that invest in public equities that GPME is large and statistically significant while CAPM alpha is virtually zero. Our empirical results suggest that GPME does not

appropriately represent the marginal utility gain from an incremental investment in PE for investors with access to secondary markets.

The buyout indices we construct have a number of potential uses for investors. Better estimates of PE risk and return should affect the optimal portfolio decisions of investors when deciding on the allocation to PE in their portfolios. In addition, the indices can be used to provide more accurate valuations of stakes in PE funds that investors hold in a manner similar to the "matrix pricing" approach commonly used to price illiquid bonds. Appendix G describes such an approach and provides annual estimates of market values for 2002 to 2014 vintage funds. Our results suggest that the use of NAV for valuation, as done by most LPs, can be misleading, and that NAVs often substantially misstate the value of an investor's PE holdings. For example, the market-to-book ratios of funds during the financial crisis reached as low as 0.60 and were as large as 1.4 to 1.5 in the years coming out of the crisis. Improving these valuations is likely to affect investors' decisions about both the portfolio allocations and the amount they spend from their invested assets.

Undoubtedly, there are uses for the indices we have not discussed in this paper. For example, one could design derivative contracts based on an index of PE returns. These derivatives could potentially be useful to LPs who wish to hedge risks in their portfolios, much like "buy-and-hold" corporate bond investors use credit and interest rate swaps to regularly optimize and rebalance their portfolios without having to trade the underlying bonds. Better indices of PE performance such as those presented here clearly have much to offer the PE community.

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Appendix A. Implicit Betas in NPV and GPME

This appendix shows how NPV and GPME can be written in terms of pre-determined discount rates and highlights the implicit betas in these valuation measures. To start, note that for any random payoff, Z_{t+j},

$$E_t[M_{t:t+j}Z_{t+j}] - Cov_t[M_{t:t+j}, Z_{t+j}] = E_t[M_{t:t+j}]E_t[Z_{t+j}], \tag{A.1}$$

which implies

$$\frac{1}{E_t[M_{t:t+j}]} - \left(\frac{1}{E_t[M_{t:t+j}]}\right) Cov_t[M_{t:t+j}, R_{z,t+j}] = \frac{E_t[Z_{t+j}]}{E_t[M_{t:t+j}Z_{t+j}]}, \tag{A.2}$$

where $R_{z,t+j}$ denotes the return for payoff Z_{t+j} at the zero-NPV price,

$$R_{z,t+j} = \frac{Z_{t+j}}{E_t[M_{t:t+j}Z_{t+j}]}. \tag{A.3}$$

But then assuming the SDF $M_{t:t+j}$ prices the risk-free asset, we have

$$E_t[M_{t:t+j}Z_{t+j}] = \frac{E_t[Z_{t+j}]}{R_{f,t:t+j} - \lambda_{t(t+j)}\beta_{z,t(t+j)}}, \tag{A.4}$$

where

$$R_{f,t:t+j} = \frac{1}{E_t[M_{t:t+j}]},$$

$$\lambda_{t(t+j)} = \frac{Var_t(M_{t:t+j})}{E_t[M_{t:t+j}]}, \tag{A.5}$$

$$\beta_{z,t(t+j)} = \frac{Cov_t[M_{t:t+j}, R_{z,t+j}]}{Var_t(M_{t:t+j})}.$$

It follows that if we set $Z_{t+1} = E_t[X_{i,t+1} + P_{i,t+1}]$, then

$$NPV_{i,t} = \frac{E_t[X_{i,t+1} + P_{i,t+1}]}{R_{f,t:t+1} - \lambda_{t(t+1)}\beta_{it}} - p_{i,t} \tag{A.6}$$

where β_{it} represents the total-return beta given by

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$$\beta_{i,t} = \frac{Cov_t[M_{t:t+1}, R_{i,t+1}]}{Var_t(M_{t:t+j})}, \tag{A.8}$$

and the total return, $R_{i,t+1}$, is

$$R_{i,t+1} = \frac{E_t[X_{i,t+1} + P_{i,t+1}]}{E_t[M_{t:t+1}(X_{i,t+1} + P_{i,t+1})]}. \tag{A.9}$$

Similarly, it follows that if we set $Z_{t+j} = E_t[X_{i,t+1}]$, then GPME can be expressed as the present value of all future cash flows,

$$GPME_{i,t} = \sum_{j=1}^{T-t} \frac{E_t(X_{i,t+j})}{R_{f,t:t+j} - \lambda_{t(t+j)}\mathcal{B}_{i,t(t+j)}} - p_{i,t}, \tag{A.10}$$

where $\mathcal{B}_{i,t(t+j)}$ represents the cash-flow-yield beta given by

$$\mathcal{B}_{it(t+1)} = \frac{Cov_t[M_{t:t+j}, Y_{i,t+j}]}{Var_t(M_{t:t+j})}, \tag{A.12}$$

and the cash-flow yield is

$$Y_{i,t+j} = \frac{X_{i,t+j}}{E_t[M_{t:t+j}X_{i,t+j}]}. \tag{A.13}$$

Appendix B. Returns from Book-to-Market Ratios

In this appendix we demonstrate the derivation of equation (26) of the paper. Let the log return on a given PE fund be defined as

$$r_{t+1} = \ln\left(\frac{P_{t+1} + X_{t+1}}{P_t}\right), \tag{B.1}$$

where P_t is the market value of the portfolio at time t, and X_t represents cash flows (total distributions minus capital calls) for the portfolio from $t-1$ to t. Equation (B.1) corresponds to a single fund but here we omit fund-level subscripts for notational ease. Note that X_t may be positive, negative, or zero. Similarly, define the log NAV-based return as

$$n_{t+1} = \ln\left(\frac{NAV_{t+1} + X_{t+1}}{NAV_t}\right). \tag{B.2}$$

If $X_{t+1} \neq 0$, then it follows that the difference between these two return measures can be written as

$$
\begin{aligned}
n_{t+1} - r_{t+1} &= \ln\left(\frac{NAV_{t+1} + X_{t+1}}{|X_{t+1}|}\right) - \ln\left(\frac{P_{t+1} + X_{t+1}}{|X_{t+1}|}\right) - \theta_t \\
&= \ln\left(\exp(\eta_{n,t+1}) + \gamma_{t+1}\right) - \ln\left(\exp(\eta_{r,t+1}) + \gamma_{t+1}\right) - \theta_t,
\end{aligned} \tag{B.3}
$$

where

$$
\begin{aligned}
\eta_{n,t+1} &= \ln\left(\frac{NAV_{t+1}}{|X_{t+1}|}\right), \\
\eta_{r,t+1} &= \ln\left(\frac{P_{t+1}}{|X_{t+1}|}\right), \\
\theta_t &= \ln\left(\frac{NAV_t}{P_t}\right),
\end{aligned} \tag{B.4}
$$

and

$$\gamma_{t+1} = \pm 1,$$

where the sign of γ_{t+1} depends on the sign of X_{t+1}. Vuolteenaho (2002) derives a similar approximation scaling by dividends. Since cash flows for PE can be negative, unlike common stock dividends, we scale by the absolute value of net cash flows, otherwise η_{nt} and η_{rt} are not well defined. A first-order Taylor series approximation of (B.3) centered around η_0 implies

$$n_{t+1} - r_{t+1} \approx \rho_{t+1}(\eta_{nt+1} - \eta_{rt+1}) - \theta_t$$

$$= \rho_{t+1}\theta_{t+1} - \theta_t, \tag{B.5}$$

where the constant of approximation ρ_{t+1} is given by

$$\rho_{t+1} = \frac{\exp(\eta_0)}{\exp(\eta_0) + \gamma_{t+1}}. \tag{B.6}$$

Alternatively, if $X_{t+1} = 0$, then the second row of (B.5) holds exactly at $\rho_{t+1} = 1$. Hence, we allow the constant of approximation to change depending on whether cash flows are positive, negative, or zero. Equation (B.5) implies

$$r_{t+1} \approx n_{t+1} - \rho_{t+1}(\eta_{nt+1} - \eta_{rt+1}) + \theta_t, \tag{B.7}$$

which is equation (26) of the paper.

The median value of η_{nt} across all fund-quarters in our sample is about 2.68, corresponding to

$$\rho_{t+1} = \begin{cases} 0.94 \text{ for } \gamma_{t+1} = 1 \\ 1.07 \text{ for } \gamma_{t+1} = -1. \end{cases} \tag{B.8}$$

Our results are quite robust to this specification and are markedly similar if we specify ρ_{t+1} to be any value in the range of 0.90 to 0.99 for observations with $\gamma_{t+1} = 1$, and in the range of 1.01 to 1.10 for observations with $\gamma_{t+1} = -1$. For our main results we specify ρ_{t+1} as in (B.8).

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Appendix C. Log Returns

In this appendix we demonstrate that the realized log return on an equally weighted portfolio is the average log return across assets in the portfolio to a first-order approximation. Let $R_1, \ldots, R_N$ denote the gross returns on N assets over a given period. The realized log equally weighted portfolio return is

$$r_p = \ln\left[\frac{1}{N}\sum_{i=1}^{N} R_i\right] = \ln\left[\sum_{i=1}^{N} e^{r_i}\right] - \ln(N), \tag{C.5}$$

where $r_i = \ln(R_i)$. Given that $r_1 = \cdots = r_n = 0$, then $r_p = 0$ and

$$\left.\frac{\partial r_p}{\partial r_i}\right|_{r_i=0} = \frac{1}{N}. \tag{C.6}$$

Hence, to a first-order approximation around the point $r_1 = r_2 = \cdots = r_N = 0$,

$$r_p \approx \frac{1}{N}\sum_{i=1}^{N} r_i. \tag{C.7}$$

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Appendix D. Dimson-Adjusted Volatility

In this appendix we present a method to bias-adjust volatility when observed returns can be characterized by the model of Dimson (1979), who presumes that securities trade intermittently at the ends of specified periods. Similar to Dimson (1979), assume that observed index returns, $\hat{r}_t$, may be written as

$$\hat{r}_t = \sum_{j=0}^{l} \lambda_j r_{t-j} + u_t, \tag{D.1}$$

where r_t represents the i.i.d. "true" portfolio return based on end-of-quarter values and u_t is a mean-zero i.i.d. error term. From (D.1) it follows that

$$Cov(\hat{r}_t, \hat{r}_{t-1}) = \sum_{j=1}^{l} \lambda_j \lambda_{j-1} Var(r_t), \tag{D.2}$$

or rather,

$$Var(r_t) = \frac{Cov(\hat{r}_t, \hat{r}_{t-1})}{\sum_{j=1}^{l} \lambda_j \lambda_{j-1}}. \tag{D.3}$$

Let a and β denote the parameters of a linear projection of r_t on the contemporaneous market return, $r_{m,t}$. We can then write

$$\hat{r}_t = c + \sum_{j=0}^{l} \psi_j r_{m,t-j} + \varepsilon_t, \tag{D.4}$$

where c is a constant, $\psi_j = \lambda_j \beta$, and $\varepsilon_t = \sum_{j=0}^{l} \epsilon_{t-j} + u_t$ is i.i.d. and mean zero. Dimson (1979) assumes $\sum_{j=1}^{l} \lambda_j = 1$, which implies $\sum_{j=1}^{l} \psi_j = \beta$. To estimate the variance us in (D.3), we jointly estimate $Cov(r_t, r_{t-1})$ and $\psi_0, \dots, \psi_m$ by GMM. We then estimate λ_j as $\lambda_j = \psi_j / \sum_{j=0}^{l} \psi_j$ and compute the variance by scaling the auto-covariance as in (D.3). We use this volatility adjustment when computing Sharpe ratios and correlation as well. We compute standard errors for $Var(r_t)$ from the GMM covariance matrix of estimated parameters via the delta method.

Appendix E: Data Details

In this appendix we describe how we clean our data. From the transactions data we first pull all records for which their "detailed strategy" is classified as "Buyout." We then identify unique funds by their fund names, hand checking fund names that appear similar. We omit funds labeled as parallel funds, feeder funds, annex funds, sub funds, top-up funds, duplicate funds, co-investment funds, supplemental funds, and side-cars. We then clean the transactions data as follows:

1) Eliminate all funds with a total commitment less than $500M. (Smaller funds may be unique in terms of secondary-market liquidity, asymmetric information, and other features that temper their ability to represent the performance of PE as an asset class.)

2) Eliminate transactions with a price less than zero. (These are likely data errors.)

3) Eliminate transactions with a NAV less than zero. (These are likely data errors.)

4) Eliminate transactions that have the same price for every fund in the portfolio transaction. (These are portfolio transactions, with no price discrepancy between individual funds. Including such funds may introduce bias in our pricing model estimates.)

5) Eliminate transactions for which the total amount committed by the seller minus the unfunded commitment is less than zero. (These are likely data errors.)

6) Eliminate transactions for which the total capital committed is less than or equal to zero. (These are likely data errors.)

7) Eliminate transactions for which the fund name is missing. (May be indicative of data errors for this record.)

8) If multiple transactions occur on the most recent transaction date for a given fund/quarter, use only the transaction based on the highest total commitment. (We need to pick one transaction to represent the fund price for the quarter, and it seems reasonable to assume that the transaction with the highest commitment is the most representative.)

9) If multiple transaction records exist with the same fund name and commitment on the most recent transaction date for a given fund-quarter, choose one of these transactions at random as

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the transaction that represents the end-of-quarter transaction price. (We need to pick one transaction to represent the fund price for the quarter,)

10) Eliminate all remaining transactions for which the price, as a percent of NAV, is greater than three standard deviations away from the mean price across funds for a given quarter. (These are likely data errors, or simply not representative.)

After pulling data from Preqin for funds with "category_type" equal to "Buyout," we clean the data as follows:

1) Eliminate any fund-quarters for which $NAV_{i,t-1} = 0$, or the NAV-based return is otherwise missing. Note that we retain records for which $NAV_{i,t} = 0$. Once NAV hits zero, however, we no longer include the fund in the sample. (We cannot compute a return with zero in the denominator.)

2) Eliminate stale NAVs, those with a report date prior to 30 days before the end of each quarter. (Doing so eliminates nonsynchronicity in NAVs.)

3) Identify fund-quarters for which the NAV-based return is greater than three standard deviations from the mean across all funds for a given quarter. These returns appear to be inconsistent with reported IRRs from Preqin. (These are likely data errors.)

We then merge our Preqin data with the explanatory variables described in Table II, and then merge these data with the cleaned transactions data set. To merge the transaction and cash flow data, we first identify funds with identical fund names in the two databases and designate these as a match. We then identify fund names in the transaction and Preqin data that are "similar" and that also have the same vintage. Fund names A and B are considered similar if fund name A contains the first five characters of fund name B anywhere in the fund name string or vice-versa. We then hand check this list to determine which funds match. After merging we have data on 839 transactions during our sample period from March 2006 through June 2018.

Appendix F: Creating Artificial Funds that Invest in Public Equities

To create artificial PE funds that invest in public equities, we first identify the timing of flows for a "representative fund" that includes all funds in our secondary market-based PE index. The index includes funds with recorded cash flows in Preqin from 2006 to 2018 with vintages spanning from 1988 to 2018. Using this population of funds, we scale all fund flows to a $1 commitment, align flows in fund inception time (as if they all made their initial capital call at the same time), and calculate the average net flows across funds each quarter from fund inception to liquidation. We use NAV as a terminal value for funds not yet liquidated by the end of the sample, and eliminate funds with an initial capital call after December 31, 2016. These average flows represent the flows of our representative fund and are illustrated in Figure 3. The initial capital call in our sample averages about 9 cents and average drawdowns in subsequent quarters fall in the range of 3 to 5 cents. Average net flows first turn positive in quarter 14 when funds are almost four years old, and climb to a peak of 5.02 cents at quarter 28 when funds are almost seven years old. Net flows then slowly decline to zero. The longest-lasting fund in our sample paid out its last distribution in the 87th quarter following its initial capital call. The average fund in our sample lasts between nine and 10 years.

Using the relative timing and amounts of flows of the representative fund, we then use the approach of Korteweg and Nagel (2016) to create the cash flows for a series of benchmark funds to estimate the parameters of the SDF to be used in calculating GPME. We first sort stocks into size deciles each month. Then, for each decile, we create the cash flows for 59 artificial funds that make capital calls and invest the acquired capital into the assigned size decile portfolio. Funds start with an initial cash flow of zero at the end of June and December of each year and initial capital calls occur at the end of March and September. For example, the first fund makes its first capital call at the end of March 1980, the second fund makes its first capital call at the end of September 1980, and so forth, with the last (59th) fund making its first capital call at the end of March 2009. Following the algorithm, capital calls each quarter are identical to those of the representative fund in fund inception time. If the representative fund makes a payout at the end of quarter q in fund inception time, the artificial funds also make payouts at the end of quarter q equal to the sum of two components. The first component is equal to the return accumulated since the last quarter in

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which a call or distribution was made. The second component is equal to a fraction, f_q, of the remaining capital under management,

$$f_q = \min\left(\frac{q-p}{40-p}, 1\right),$$ (8)

where p is the time of the most recent payout prior to quarter q, measured in fund inception time quarters. If the payout at quarter q is the fund's first payout, then $p = 0$. This assumption sets the life of each artificial fund to 10 years, and for our setup implies that the 59th fund pays its final distribution in December 2018. We proceed in this manner to create the overlapping cash flows for 59 artificial funds that invest capital in their assigned size-decile portfolio, and then repeat this exercise for all 10 deciles.

Appendix G. An Application: "Matrix Pricing" of Private Equity Funds

Most funds are valued by LPs at the NAV, which can deviate substantially from the best available estimate of the fund's underlying value. These valuations are used for a number of purposes by investors in funds, including portfolio allocation decisions across asset classes, and spending decisions, which are usually set by investors as a fixed percentage of a portfolio's assessed value. Examples of this policy approach include universities and foundations. More accurate pricing of LP stakes in PE funds may improve the ability of investors to make investment decisions since it may lead to portfolio allocations and spending rules corresponding to better estimates of the underlying values of an institution's PE investments.

One possible approach to value stakes in PE funds more accurately is to follow a procedure similar to "Matrix Pricing," commonly used to price bonds, by which the prices of bonds that do not trade are determined based on the prices of bonds that do. The idea is that the same fundamentals affect similar bonds in the same manner, so prices of bonds that do not trade likely move approximately the same amount as prices of similar bonds that do trade. Since PE funds that invest in one type of asset are likely affected by a number of the same shocks to their fundamentals, they can be priced using comparable methods with transactions-based indices.

For any fund, the fund's history of quarterly cash inflows and outflows can be combined with the quarterly returns of the hedonic indices to calculate market values. Beginning at the end of some chosen quarter t, we set fund value for fund i, $V_{i,t}$, equal to NAV. For all subsequent quarters we estimate fund value as

$$V_{i,t} = V_{i,t-1}(1 + r_{H,t}) + C_{i,t} - D_{i,t}, \tag{G.1}$$

where C_t and D_t denote capital calls and distributions between times t and $t + 1$, and $r_{H,t}$ represents the return on our hedonic transaction-based index.

We perform this calculation for each fund in our sample for every quarter in our sample period (2006 to 2018). One version of our hedonic index is formed using transactions from funds that are between four and nine years old, and we use this version of the index to calculate market values for four- to nine-

year-old funds. Following the procedure described above, we set market value equal to NAV at the end of the fourth year following each fund's vintage year and iterate forward to identify market values at the end of subsequent quarters.[22] We report year-end aggregate market-to-book ratios by vintage in Appendix Table HII by summing year-end market values for each fund for a given vintage and dividing by the sum of year-end NAVs for the same set of funds.[23]

Aggregate market-to-book ratios for each vintage over time are reported in the bottom of Appendix Table HII. Market-to-book ratios are considerably lower during the financial crisis, ranging between 0.59 and 0.76 in 2008 for the three vintages that were old enough for our hedonic estimation. Funds that invested out of 2007 and 2008 vintage funds did so at lower valuations, and therefore have high average market-to-book ratios in subsequent years when markets recovered. For example, these estimates indicate that by the end of 2016, a 2008 buyout fund has a NAV that is understated by 28% relative to its market value.

Individual funds could mark their values to market using the hedonic approach in one of two ways. The most accurate approach would be to generate fund-specific market values using the estimated coefficients from our Heckman sample selection model applied to the fund's attributes. A simpler approach, but one that nonetheless represents a substantial improvement over using NAVs, is to multiply the NAV of each of LP's investments by the average market-to-book ratio of the industry. For example, an LP would multiply the NAV of each four- to nine-year-old fund in his or her portfolio by the appropriate ratio from Appendix Table HII. For younger funds, the deviation between NAV and market value is likely to be smaller but could be estimated using the estimated coefficients from our sample selection model. Tail-end funds will have only a few portfolio companies left and their values will vary depending on the fortunes of these particular investments. As such, this approach is likely to be less useful for valuing these funds.

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[22] For the 2005 vintage, we set NAV equal to market value at the end of year three due to irregularities in reported NAVs associated with the financial crisis.

[23] The 2018 market-to-book ratio is reported as of Q2 because of data availability.

Appendix H: Appendix Tables

Table HI. Robustness Checks

In this table we report main index parameters of interest for different versions of the model and for different methods of estimating index parameters. All expected returns and alphas are annualized. In Panel A we report index parameters for log returns after adjusting alpha as in Cochrane (2004) and Axelson, Sorensen, and Stromberg (2014). In a regression of log excess index returns on log excess market returns, let δ denote the intercept. The adjusted alpha is given by $\alpha = \delta + [(1/2)\sigma_I^2 - (1/2)\sigma_M^2\beta(1 - \beta)]$, where σ_I^2 denotes the Dimson-adjusted index variance using log returns (see Appendix D), σ_M^2 denotes the market variance, and β denotes the Dimson-adjusted index beta using one lag and log returns. All other panels report results using simple (exponentiated) returns after Dimson adjusting as discussed in the text and Appendix D. In Panel B we eliminate insignificant variables from the pricing equation as reported in columns (3) and (4) of Table II before constructing the index and estimating index parameters. In Panels C and D we replace state variables in both the pricing and selection equation with quarterly fixed effects. In Panel C we use the estimated fixed effects from the prior quarter for the two quarters with no transactions (Q3/2007 and Q1/2013). In Panel D we delete the two quarters with missing transactions. This leads us to lose four index return observations. In Panels E and F we replace state variables in both the pricing and selection equations with quarterly fixed effects and eliminate all other variables from the pricing equation. In Panel E we use the estimated FE from the prior quarter for the two quarters with no transactions. In Panel F we delete the two quarters with missing transactions. Standard errors are estimated by GMM using the Newey-West (1987) estimate of the spectral density matrix, along with the delta method. Significance at the 1%, 5%, and 10% level is indicated by ***, **, and *, respectively.

	Panel A. Log Returns (Adjusted Alpha)				Panel B. Narrow Pricing Eq I			
	All Ages		4-9 Yrs Old		All Ages		4-9 Yrs Old	
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
E[r]	0.08	(0.8)	0.16	(1.7)*	0.12	(1.4)	0.18	(2.1)
beta	1.95	(6.6)***	1.86	(7.3)***	1.76	(8.0)***	1.73	(7.8)
alpha	-0.02	(0.0)	0.00	(0.0)	-0.04	(0.8)	0.02	(0.4)
N (Pricing)	839		839		839		839	
N (Selection)	15,367		15,367		15,367		15,367	
N (Index)	49		49		49		49	

	Panel C. Quarter Fixed Effects I				Panel D. Quarter Fixed Effects II			
	All Ages		4-9 Yrs Old		All Ages		4-9 Yrs Old	
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
E[r]	0.16	(1.0)	0.23	(1.4)	0.16	(0.9)	0.22	(1.2)
beta	1.64	(3.0)***	1.63	(3.0)***	1.77	(3.4)	1.77	(3.4)***
alpha	0.01	(0.1)	0.08	(0.5)	0.00	(0.0)	0.06	(0.4)
N (Pricing)	839		839		839		839	
N (Selection)	15,367		15,367		14,835		14,835	
N (Index)	49		49		45		45	

	Panel E. Quarter Fixed Effects III				Panel F. Quarter Fixed Effects IV			
	All Ages		4-9 Yrs Old		All Ages		4-9 Yrs Old	
	(1)	(2)	(3)	(4)	(1)	(2)		(3)	(4)
E[r]	0.17	(1.0)	0.22	(1.3)	0.18	(1.0)		0.23	(1.2)
beta	1.98	(3.8)***	1.96	(3.7)***	2.05	(3.9)	***	2.04	(3.8)***
alpha	-0.01	(0.1)	0.04	(0.3)	0.00	(0.0)		0.05	(0.3)
N (Pricing)	839		839		839			839	
N (Selection)	15,367		15,367		14,835			14,835	
N (Index)	49		49		45			45	

Table HII. **Aggregate Market-to-Book Ratios of Private Equity Investments**

This table reports year-end average market-to-book ratios. Market values for each fund are calculated using the following procedure. We begin by assuming that the market value of the fund is equal to NAV in years one through four of the fund's life. We then calculate the market value each quarter from years five to nine for fund i using the formula:

$$V_{i,t} = V_{i,t-1}(1 + r_{H,t}) + C_{i,t} - D_{i,t},$$

where $V_{i,t}$ is equal to the NAV of fund i at time t, C_t and D_t denote capital calls and distributions between times t and $t+1$, and $r_{H,t}$ represents the return on our hedonic transaction-based index. For the first quarter in year five, we use NAV as the preceding quarter's market value. The aggregate market-to-book ratio reported in this table is calculated as the sum of the individual fund's market value within each quarter divided by the sum of the individual fund's NAV in each quarter. We report the resulting market-to-book ratio for Q4 of each year, with the exception of 2018, where we report values as of Q2 due to data limitations.

	Vintage Year												
	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
2006	1.07	--	--	--	--	--	--	--	--	--	--	--	--
2007	0.83	0.93	--	--	--	--	--	--	--	--	--	--	--
2008	0.59	0.60	0.76	--	--	--	--	--	--	--	--	--	--
2009	0.98	0.82	1.02	0.78	--	--	--	--	--	--	--	--	--
2010	1.16	0.95	1.28	1.11	1.10	--	--	--	--	--	--	--	--
2011	1.35	0.95	1.32	1.31	1.06	0.98	--	--	--	--	--	--	--
2012	--	1.11	1.73	1.34	1.19	1.13	1.01	--	--	--	--	--	--
2013	--	--	2.59	1.71	1.49	1.39	1.22	1.04	--	--	--	--	--
2014	--	--	--	2.58	1.51	1.28	1.07	0.86	0.88	--	--	--	--
2015	--	--	--	--	1.44	1.30	1.02	0.70	0.79	0.95	--	--	--
2016	--	--	--	--	--	1.81	1.28	0.77	1.05	1.20	0.97	--	--
2017	--	--	--	--	--	--	1.60	0.70	1.14	1.30	0.96	1.14	--
2018	--	--	--	--	--	--	--	0.67	1.40	1.42	0.95	1.16	0.99
Average	1.00	0.89	1.45	1.47	1.30	1.31	1.20	0.79	1.05	1.22	0.96	1.15	0.99
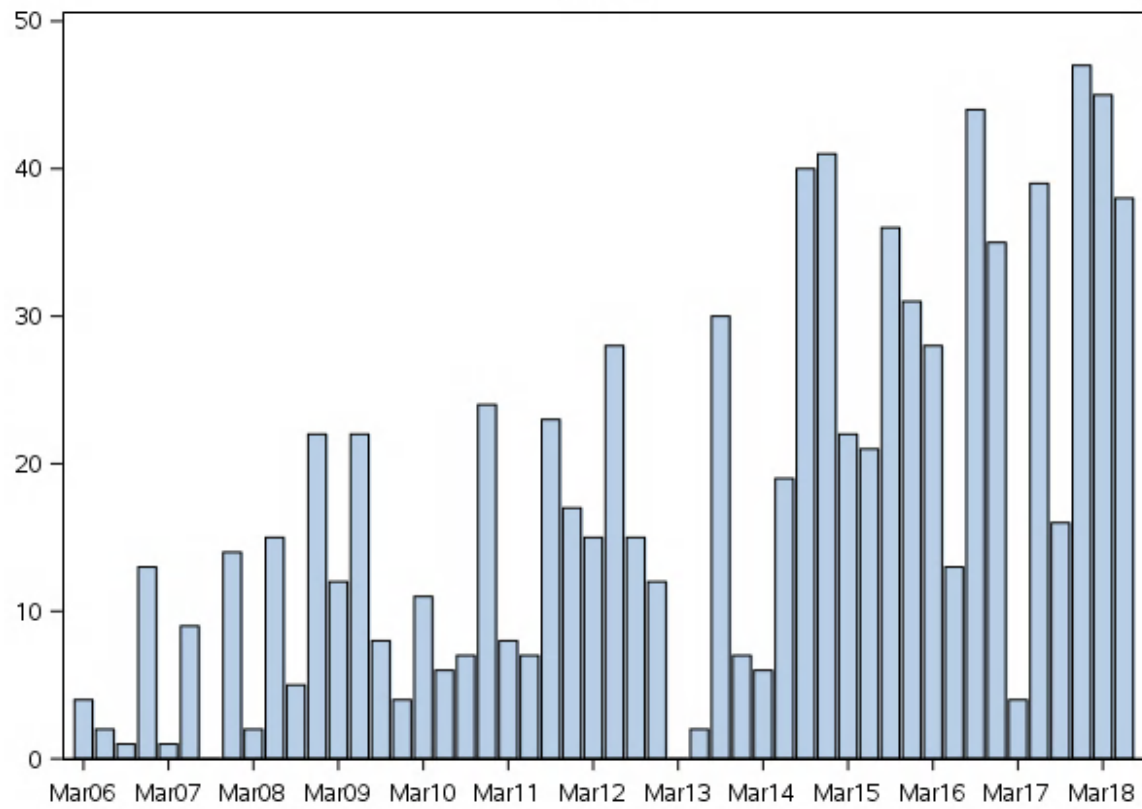
Figure 1. Number of transactions per quarter. In this figure we plot the number of PE transactions we observe per quarter after cleaning the data as described in the paper and Appendix E.
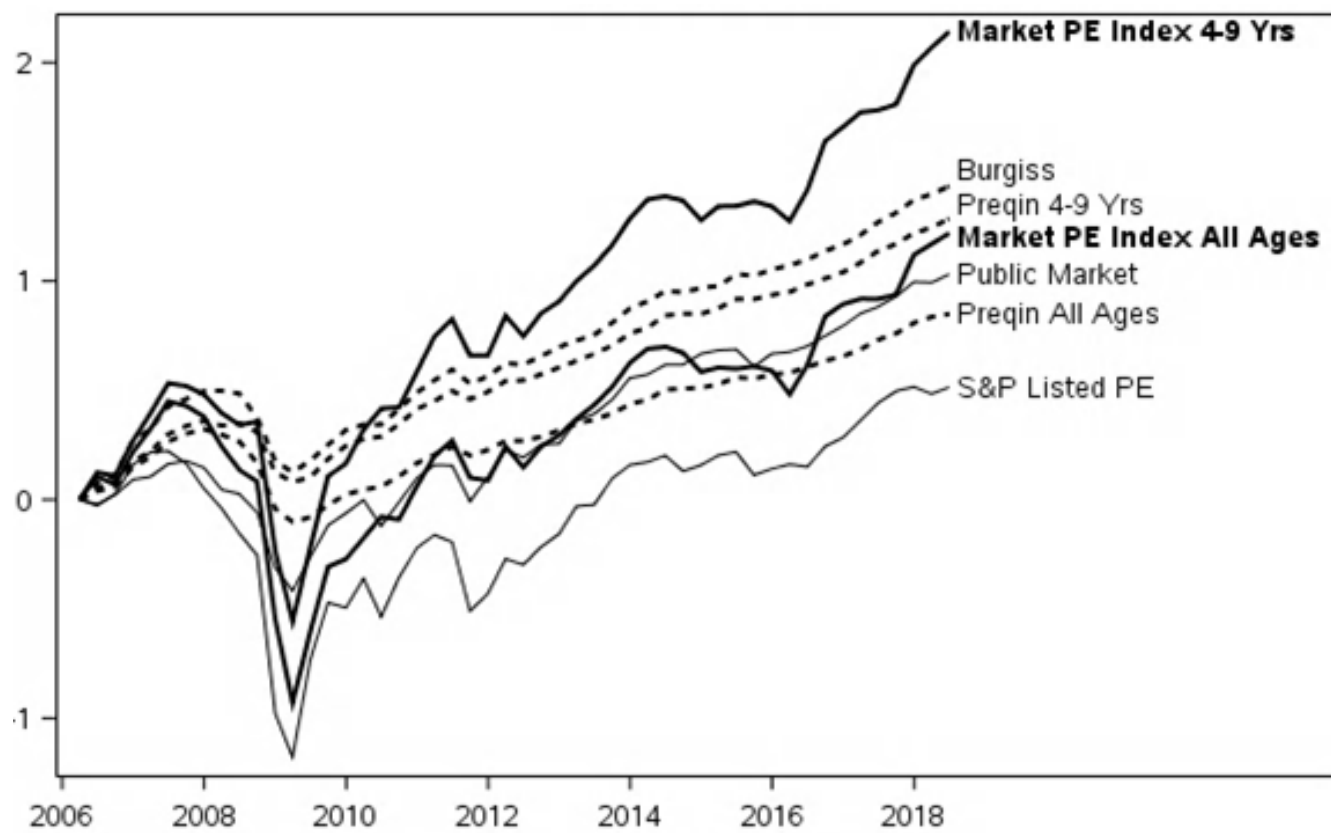
Figure 2. Indices over time. This figure illustrates the value of investing $1 in various indices at the beginning of 2006 as labeled. The Market PE indices are the indices based on secondary market transactions. The Public Market index is based on the public market return as posted on Ken French's website. We build the Preqin indices based on NAVs reported in Preqin. The Burgiss index is a NAV-based buyout index. The S&P Listed Private Equity Index is an index comprised of publicly traded PE funds. The chart uses a log scale for the vertical axis.
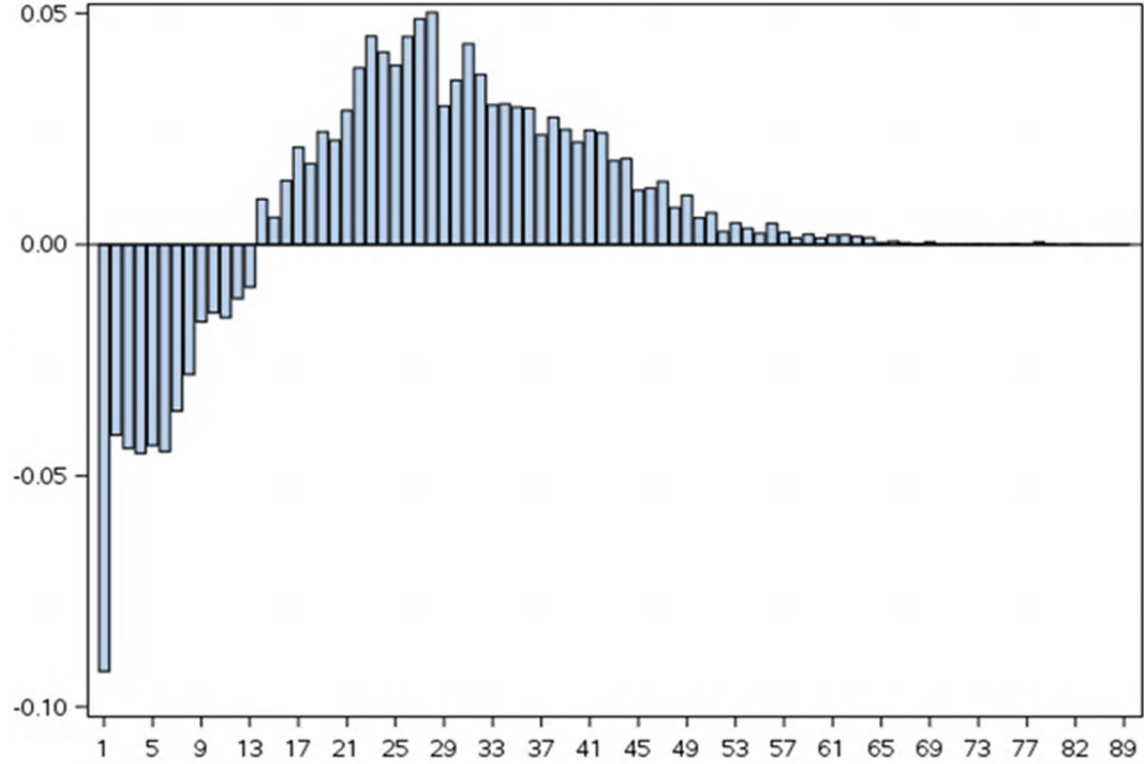
Figure 3. Cash flows for the representative fund. This figure illustrates the cash flows of the representative fund we use to create synthetic funds that invest in stock deciles. These cash flows are the average cash flows of all funds in our index based on fund inception time. We build our index using funds with recorded cash flows in Preqin from 2006 to 2018 with vintages spanning from 1988 to 2016. We scale all fund flows to a $1 commitment, align flows in fund inception time (as if they all made their initial capital call at the same moment), and calculate the average net flows across funds each quarter from fund inception to liquidation. We use NAV as a terminal value for funds not yet liquidated by the end of the sample.

Table I Summary Statistics

This table reports summary statistics for the variables used in our Heckman model. The first column reports statistics for records in our data with transactions. The second column reports statistics for records without transactions. The symbol θ_{it} represents the log book-to-market ratio of fund i at time t, $\log(size_{it})$ is the log of total capital committed for fund i at time t, Age_{it} is the age of fund i at time t, $PME_{i,t}$ is the public market equivalent of Kaplan and Schoar (2005) for fund i at time t, $PF_{i,t}$ is the fraction of LPs that are pension funds for fund i at time t, $\log(BM_t)$ is the log value-weighted book-to-market ratio at time t using stocks with a market cap less than \$500 million and share code 10 or 11 in CRSP, $IAUM_t$ is total assets under management in PE at time t scaled by the total number of firms that have between 20 and 500 employees at the end of the prior year as reported by the US Census, Ted_t is the TED spread at time t measured as the spread between three-month LIBOR based on U.S. dollars and three-month Treasury yields, $N\ per\ Quarter$ is the number of observations each quarter in the sample. "Mean," "Median," and "Stdev" represent the average, median, and standard deviation across time and across all funds (if applicable), "Q1" is the 25th percentile, and "Q3" is the 75th percentile.

		Transactions	No Transaction
		(1)	(2)
$\theta_{i,t}$ (dep variable)	Mean	0.19	
	Median	0.10	
	Stdev	0.42	
$\log(size_{i,t})$	Mean	21.95	21.28
	Median	21.99	21.13
	Stdev	0.96	0.87
$Age_{i,t}$	Mean	8.61	6.39
	Median	9.00	6.00
	Stdev	4.05	4.15
$PME_{i,t}$	Mean	1.15	1.13
	Median	1.10	1.06
	Stdev	0.36	0.37
PF_{it}	Mean	0.53	0.56
	Median	0.55	0.57
	Stdev	0.17	0.20
$\log(BM_t)$	Mean	-0.22	-0.25
	Median	-0.22	-0.23
	Stdev	0.24	0.26
$IAUM_t$	Mean	3.29	3.28
	Median	3.53	3.53
	Stdev	0.75	0.75
TED_t	Mean	0.48	0.47
	Median	0.33	0.32
	Stdev	0.49	0.46
$N\,per\,Quarter$	Mean	17.48	290.56
	Q1	7	237
	Q3	27	353
$Quarters$		48	50
N		839	14,528

Table II Sample Selection Model Estimates

This table reports estimates of parameters for the Heckman (1979) sample selection model discussed in the paper. Columns (1) and (2) report estimates and t-statistics of parameters for the selection equation while columns (3) and (4) report estimates and t-statistics for the pricing equation. "Heckman" refers to the sample selection model, while "OLS" indicates the pricing model is estimated by simple OLS with no selection equation. Variables are described in the heading for Table I. We estimate standard errors for the Heckman model using a panel bootstrap, to account for the inverse Mills ratio being a generated regressor, and cluster by time. We estimate White (1980) standard errors for the OLS model and cluster by time. Significance at the 1%, 5%, and 10% levels is indicated by ***, **, and *, respectively.

	Heckman				OLS	
	Selection		Pricing		Pricing	
	estimate	(t-stat)	estimate	(t-stat)	estimate	(t-stat)
	(1)	(2)	(3)	(4)	(5)	(6)
log(BM_t)	-0.03	-(0.1)	0.79	(3.8)***	0.79	(5.1)***
TED_t	0.31	(2.0)**	-0.05	-(0.4)	-0.06	-(0.9)
$IAUM_t$	0.20	(2.2)**	-0.20	-(2.4)**	-0.21	-(3.3)***
log($size_{i,t}$)	0.40	(13.4)***	0.00	(0.0)	-0.02	-(1.9)*
$I(Age<4)$	-0.50	-(4.1)***	0.15	(0.8)	0.18	(2.6)***
$I(Age>9)$	0.33	(4.6)***	0.20	(1.7)*	0.18	(4.0)***
$PME_{i,t}$	-0.13	-(2.0)**	-0.08	-(1.5)	-0.07	-(2.2)**
$PF_{i,t}$	-0.32	-(2.8)***				
Inv Mills Ratio			0.06	(0.3)		
Intercept	-10.66	-(14.7)***	1.05	(0.3)	1.65	(5.6)***
N (selected)	839		839		839	
N (not selected)	14,528					

Table III Predicting Index Returns Using Lagged Market Returns

This table reports estimates from regressions of index returns on contemporaneous and lagged market returns. The transaction index returns are created using the sample selection model as described in the paper. The S&P Listed Private Equity Index is an index comprised of publicly traded PE funds. We build the *Preqin* indices based on NAVs reported in Preqin using funds in the transaction-based indices. The Burgiss index is a NAV-based buyout index.

| | Transactions | | | | S&P Listed | | Preqin NAV | | | | Burgiss | |
| | All Ages | | 4-9 Yrs Old | | | | All Ages | | 4-9 Yrs Old | | | |
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
a	-0.01	-(0.79)	0.01	(1.35)	-0.01	-(1.36)	-0.00	-(0.88)	0.01	(2.11)**	0.01	(1.25)
b_0	1.24	(4.78)***	1.22	(5.01)***	1.71	(7.54)***	0.42	(5.95)***	0.40	(7.30)***	0.50	(7.70)***
b_1	0.60	(4.33)***	0.59	(4.53)***	0.01	(0.07)	0.15	(5.48)***	0.14	(6.12)***	0.12	(3.62)***
b_2	-0.27	-(1.20)	-0.32	-(1.46)	-0.32	-(1.56)	0.10	(3.31)***	0.07	(3.26)***	0.11	(2.74)***
b_3	0.24	(1.93)*	0.22	(1.55)	0.03	(0.23)	0.12	(2.68)***	0.09	(2.13)**	0.10	(2.17)**
b_4	-0.13	-(0.86)	-0.19	-(1.39)	-0.06	-(0.75)	0.04	(1.03)	0.01	(0.19)	0.01	(0.19)
N	45		45		45		45		45		45	

Table IV Index Parameters

This table reports index parameters for buyout indices using data from 2006 to 2018. Columns (1) and (2) report results for our transactions-based indices using funds of all ages, columns (3) and (4) report results for our transactions-based indices using funds that are four to nine years old, columns (5) and (6) report results for the S&P listed index, columns (7) and (8) report results for our Preqin NAV-based indices using funds of all ages, columns (9) and (10) report results for our Preqin NAV-based indices using funds that are four to nine years old, and columns (11) and (12) report results for the Burgiss index. Moments of indices are Dimson adjusted using a number of lags based on results from Table III. "AC" represents autocorrelation and is not Dimson adjusted. We estimate standard errors for index parameters by GMM using the approach of Newey and West (1987) to estimate the spectral density matrix and the delta method. Significance at the 1%, 5%, and 10% levels is indicated by ***, **, and *, respectively.

	Transactions				S&P Listed		Preqin NAV				Burgiss	
	All Ages		4-9 Yrs Old				All Ages		4-9 Yrs Old			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
E[r]	0.14	(1.6)	0.22	(2.5)**	0.09	(1.0)	0.11	(3.4)***	0.14	(4.6)***	0.12	(3.2)***
β	1.79	(8.0)***	1.76	(8.7)***	1.74	(8.1)***	0.72	(5.7)***	0.64	(5.9)***	0.81	(5.7)***
α	-0.02	-(0.4)	0.06	(1.1)	-0.07	-(2.7)***	0.04	(1.6)	0.07	(3.3)***	0.04	(1.6)
σ	0.34	(2.8)***	0.31	(2.8)***	0.30	(4.2)***	0.17	(2.6)**	0.14	(2.5)***	0.20	(2.3)**
Sharpe	0.41	(1.3)	0.68	(1.9)*	0.27	(1.3)	0.67	(2.3)**	0.99	(2.7)***	0.60	(1.9)*
Corr w/ Mkt	0.83	(4.0)***	0.88	(3.7)***	0.90	(2.0)**	0.70	(1.7)*	0.76	(1.8)*	0.65	(1.6)
AC (unadjusted)	0.30	(0.8)	0.26	(0.7)	0.22	(0.5)	0.54	(1.4)	0.41	(0.8)	0.46	(1.3)
N (Pricing)	839		839									
N (Selection)	15,367		15,367									
N (Index)	49		49		49		49		49		49	

Table V Campbell-Shiller Regressions

This table reports the slope coefficient in Campbell-Shiller regressions as described in the paper,

$$\sum_{j=1}^{k} \bar{r}_{t+j} = a_r + \beta_r \bar{\theta}_t + w_{r,t+k}$$
$$-\sum_{j=1}^{k} \bar{n}_{t+j} = a_n + \beta_n \bar{\theta}_t + w_{n,t+k}$$
$$\varepsilon_{t+k} = a_\varepsilon + \beta_\varepsilon \bar{\theta}_t + w_{\varepsilon,t+k},$$

where $\bar{r}_{t+j}$ represents the log return based on transaction prices, $\bar{n}_{t+j}$ is the NAV-based (book-value) log return, $\bar{\theta}_t$ is the log book-to-market ratio, and the key component of ε_{t+k}, further defined in the paper, is $\bar{\theta}_{t+k}$. The data are quarterly. Panel A reports results for $k = 1$ (one quarter) while Panels B and C report results for overlapping regressions where $k = 4$ (one year) and $k = 20$ (five years). On the left side of each panel we report results using $\bar{\theta}_t$ as the sole explanatory variable. On the right side of each panel we report results that also control for $\bar{r}_t$, and $\bar{n}_t$ as explanatory variables. We compute Hansen-Hodrick (1980) standard errors that account for overlapping returns in our regressions. Significance at the 1%, 5%, and 10% levels is indicated by ***, **, and *, respectively.

	Extra Control Variables: No				Extra Control Variables: $\bar{r}_t, \bar{n}_t$		
	slope coefficient on $\bar{\theta}_t$	st. err	R^2		slope coefficient on $\bar{\theta}_t$	st. err.	R^2
Panel A. $k = 1$ quarter							
β_r	0.06	(0.12)	0.01		0.31	(0.10)	0.45
β_n	0.03	(0.04)	0.03		-0.04	(0.03)	0.40
β_ε	0.89	(0.08)***	0.75		0.71	(0.08)	0.84
N	49				49		
Panel B. $k = 4$ quarters							
β_r	0.45	(0.43)	0.07		0.65	(0.44)	0.12
β_n	0.01	(0.17)	0.00		-0.10	(0.16)	0.09
β_ε	0.83	(0.26)***	0.41		0.56	(0.25)	0.54
N	46				46		
Panel C. $k = 20$ quarters							
β_r	1.65	(0.50)***	0.54		1.67	(0.51)	0.58
β_n	-0.44	(0.29)	0.22		-0.52	(0.28)	0.31
β_ε	0.05	(0.60)	0.00		-0.25	(0.58)	0.11
N	30				30		

Table VI Performance Metrics for Synthetic Funds

This table reports performance metrics for a set of synthetic funds that invest capital in US equities. We sort stocks into deciles and, for each decile, create a set of funds that make initial capital calls at the end of June and December of each year from March 1980 through March 2009. Each fund then makes capital calls and distributions following the algorithm of Korteweg and Nagel (2016) to mimic the cash flows of a representative fund over a 10-year period, with each fund investing the acquired capital into their assigned decile portfolio and the last fund making its final distribution December 2018. For each set of funds, column (2) reports the average size of firms within each fund in 2020 $millions, column (3) reports the average IRR, and columns (4) and (5) report the GPME and corresponding J-statistic p-value. Column (6) reports the SDF alpha for each decile portfolio, where the SDF is given by equation (41) in the paper, column (7) reports the corresponding p-value of the t-statistic, column (8) reports the standard CAPM alpha for each decile portfolio, and column (9) reports the corresponding p-value of the t-statistic. Significance at the 1%, 5%, and 10% levels is indicated by ***, **, and *, respectively.

	Decile (1)	Mean Mcap $MM Y2020 (2)	IRR (3)	GPME (4)	p-value (5)	SDF alpha (6)	p-value (7)	CAPM alpha (8)	p-value (9)
	1	17.0	0.27	0.608	(0.00)***	0.099	(0.26)	0.024	(0.02)**
	2	40.7	0.10	0.201	(0.05)**	-0.009	(0.92)	-0.006	(0.46)
	3	81.6	0.10	0.193	(0.05)**	-0.009	(0.91)	-0.008	(0.24)
	4	147.0	0.10	0.308	(0.01)**	0.041	(0.71)	-0.006	(0.34)
Public Equities	5	256.5	0.11	0.237	(0.02)**	0.059	(0.62)	-0.006	(0.26)
1980-2018	6	446.9	0.12	0.209	(0.01)***	0.070	(0.54)	-0.003	(0.52)
	7	792.9	0.12	0.142	(0.01)***	0.071	(0.54)	-0.003	(0.38)
	8	1,511.3	0.13	0.113	(0.00)***	0.077	(0.49)	-0.001	(0.83)
	9	3,712.8	0.13	0.066	(0.00)***	0.062	(0.55)	0.000	(0.86)
	10	86,970.4	0.12	-0.040	(0.04)**	0.063	(0.55)	0.001	(0.42)
	N	Average of 538 Firms per Decile	59 Funds per Decile	59 Funds per Decile		157 Obs per Decile		157 Obs per Decile	